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EDITOR'S NOTE

Crossroads is an interdisciplinary, undergraduate research journal published by the Monmouth University Honors School. The contributors are Senior Honors Thesis students whose work has been chosen by the Honors Council as representing the most original, thoroughly researched, and effectively argued theses in their fields.

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Nationalism, Language, and Education Policy: France and Ukraine

Lindsay Savage

“Speech distinguishes man among the animals; languages distinguish nations from each other; one does not know where a man comes from until he has spoken.”

-Jean-Jacques Rousseau

“All nationalisms are at heart deeply concerned with names: with the most immaterial and original human invention. Those who dismiss names as a detail have never been displaced; but the peoples on the peripheries are always being displaced. That is why they insist upon their continuity—their links with their dead and the unborn.”

-John Berger

“Where ever a separate language is found, there is also a separate nation which has the right to manage its affairs...and to rule itself”

-Johann Fichte

Abstract

This article proposes that a link exists between education policies and nationalism; that is, education policies are designed and implemented with the purpose of creating a nationalist sentiment within a community. In the context of this project, one of the most important components of nationalism is the promotion of a standardized language which is used in all forms of social behavior, in the home, school, and in the workplace. This article will include a theoretical discussion of paradigms of nationalism (focus on Ernest Gellner’s theory of High Culture), and then two case studies will be presented in order to illustrate the connection between nationalism and education. The two case studies selected are: the success of the French

with the intent of creating a nationalist sentiment within a community. Nationalism is understood as the *idea of identity* within a particular community, and the desire to make this idea uniform and- most importantly- dominant. Idea of identity is understood as the shared characteristics of a specific population which, when viewed collectively, offer a definition of what, exactly, demonstrates membership to that population (and, perhaps more importantly, what does *not* characterize membership to the population). One of the most important components of nationalism is the promotion of a standardized language which is used in all forms of social behavior, in the home, school, and in the workplace. The central focus will demonstrate how the attitudes and actions of education organization leading to a systemized national language explain the influence nationalism has in creating education policy.

It is important to note that nationalism as a viable political construct does not need to have a *nation-state* attached to it. While it is often the case that nationalism is identified with an attendant nation, in some instances, communities can be highly nationalistic and not have a nation-state of their own. This is evident in the case of the Kurdish population in Iraq and Turkey who have no nation-state yet have a very definite *idea of identity*. Europe also boasts many national communities that are without nation-states. The Basques in Spain are a good example of this situation.

From the description presented, it is obvious that *language* plays a huge part in this project. While nationalism has many facets, and a shared community must have a shared history as well as shared myths and a general shared culture, in the end it is language which goes the furthest to define a community in terms of shared identity. Struggles for language supremacy in a region dominate the history of educational policies and nationalism, and hence, this paper will focus largely on how language comes into play regarding the connection between education and nationalism.

Keeping the idea of nationalism in mind will help in exploring the link to education and, most importantly, education policies. A combination of theoretical constructs and case studies, will demonstrate how policies are implemented to make education a tool of nationalism.¹

component of spreading nationalism. The standardization of language goes a long way to forming at least a coherent, functioning idea of a national collective. Gellner's theory of High Culture also explores the economic angle of nationalism. In order for an individual to function in a market dominated by a particular language and custom, that individual must learn the language. The educational system is the method of insuring economic participation by imparting knowledge of the language which is necessary for success.²

Once the theoretical dimensions are laid out, a historical overview of education's place in building and strengthening nationalism will be offered. While the portion of this paper devoted to the two case studies that have been selected will offer a more in-depth and detailed discussion of distinct national policies, this section will provide different examples of education being used to further nationalism. While the historical survey presented will be very general in its approach, this paper will attempt to give a broad understanding of how education and nationalism have come together in the past.³ The idea of education as a branch of public policy, especially in regard to national standardization will also be explored.⁴ Drawing from Gellner's theoretical framework, examples of state-run education attempting to define a language community will be provided.

After giving a general survey both of nationalist theory and its relationship to education and education policy, the two case studies will be presented; these case studies have been selected in order to provide a detailed assessment of education policies used to further the nationalist cause. The first case study will involve a nation which has already cemented its nationalist identity, while the second case study will explore the education

these two constructs together along with a focus on education has been very helpful in my study of nationalism.

² Gellner's book *Nations and Nationalism* forms a large part of the research into this paradigm.

³ Special attention will be paid to the case of the Austro-Hungarian Empire,

system will follow, including how the French language was standardized in order to create a French nationalist community. The creation of a nationally French identity is closely tied with the adoption of a single French language. France became the first example of a cohesive nation after the advent of the French Revolution⁵, and then attempted (and subsequently succeeded) to solidify its national identity by dispelling all languages other than standardized French. France, once a nation of hundreds of regional dialects, became homogenized in terms of linguistics, much of which began during the nineteenth century. I will explore in detail the part that the education system played in this standardization of language and culture, and how the French educational system has succeeded in standardizing the French language.

The second case study will focus on a state with a less developed national identity- or, rather, a state in which the concept of national identity has been oppressed by years of outside dominance. The Ukraine, for most of its recent history, was under the direct control of the Soviet Union. The power of Soviet control reached into the heart of Ukraine's policy directives and educational system, absorbing any nationalist tendencies. Ukrainian nationalism was heavily discouraged under Soviet suzerainty. This discouragement included the language. As in many of the former Soviet republics, the Russian language was prized far above any native tongue. As was the case with the Austro-Hungarian Empire, the Soviet Union attempted to quash any nationalist sentiments by creating an almost universal nationalism, that is, the ideals of the Communist party and loyalty to the Soviet Union itself. However, with the collapse of the Soviet Union, Ukraine has become an independent state, able to function in its own right as a nation with the goal of finally creating a culturally united population. With the departure of Soviet power, the Ukraine can now focus on solidifying a standardized concept of Ukrainian nationalism, including language. The effects of Soviet dominance on the Ukrainian educational system will be discussed, and then the changes wrought in Ukrainian education policy since independence will be explored.

Revolution” and other movements which have stressed Ukraine’s break with Russia. Nationalism is once again being redefined as *what one is not*, and in the case of the Ukraine (especially in terms of language and hence the standardization of education), what is Ukrainian is inherently what is not Russian.

The importance of this study lies in the effects nationalism has had on the world. Nationalism has defined much of our present society, and the effects of nationalism are directly responsible for the world in which we live today. Despite the proposed homogenization of world culture brought about by globalization, nationalism and the desire for individual communities defined by aspects such as language and shared histories is still an impelling force in world affairs and the creation of policy. In some ways, globalization may have even strengthened the cause for nationalist sentiments.⁶ Information technology and other hallmarks of a globalized world have allowed different national communities to express and communicate their inherent desire for independence with broader audiences in ways which had previously been impossible. Education is both a cause and an effect in this struggle, and the policies set by those in charge of national education are quite responsible for the direction a particular community may follow. Language, culture, history, and identity are still very important in our world. The education policies which further these concepts must be understood in order to determine where a particular nation is headed.

Literature Review

The key concepts that will be addressed can be categorized under a group of subject headings: nationalism as a theoretical construct, nationalism and its practical components within society, education as an outlet of nationalism, the history of the relationship between education and nationalism, the importance of language as an identifying factor of nationalism, and finally the examination of education and nationalism within the contexts of France and Ukraine. There are, of course, myriad secondary

European history enter into the subject matter as well, and bear significance on the findings.

Much has been written on all the topics addressed above, and it would be impossible to encompass all the literature dedicated to even one of the aforementioned concepts. The most relevant literature has been selected in order to provide the most useful knowledge base in order to construct a compelling argument. This section will provide a breakdown of the literature utilized, divided into the most appropriate subject headings in order to illustrate the connections between the scholarly works consulted.

Nationalism

Obviously, nationalism is at the very heart of this project; a solid understanding of both nationalism as a theoretical construct and as a practical element in society is therefore of the utmost importance to the arguments presented. There is a wealth of literature that deals with nationalism in all its phases, but in terms of this paper, the starting point for theoretical works examining nationalism, its origins and its effects, is Ernest Gellner. Gellner's works *Nations and Nationalism* and *Thought and Change* provide a large part of the theoretical paradigm, especially concerning education and nationalism, and the connection between the two. O'Leary's work is also consulted in attempting to streamline an understanding of Gellner's theory, along with providing some possible criticism of Gellner's work.⁷ Complementing Gellner's theory is Benedict Anderson's work *Imagined Communities*, which takes a view of nationalism that is heavily reliant on the advent of media, and the dissemination of language for transmitting information. Anderson's theory is relevant in that it stresses the importance of a strong language community as a presupposition for nationalism, and education's place in creating a dominant language.

In addition to the main theoretical constructs utilized, Anthony Smith is a particularly prolific author in the field, and his works form an important base of understanding nationalism, both in terms of its political effects and the arguments concerning its continued relevance as a global

This section starts out with Susanne Wiborg's excellent article on two different varieties of nationalism: political (as espoused by Rousseau) and cultural (with Herder as its spokesperson). These two varieties of nationalism are important to understand in regard to the case studies presented. Aside from the theoretical aspect of nationalism, historic examples of the effects of nationalism in regard to educational policy have an important place in the discussion presented by this paper, even if they are not given as much attention as the two case studies. Since the examples given are all of European nation-states, M.S. Anderson's *The Ascendancy of Europe: 1815-1914* has been consulted for the background information necessary to make some coherent sense of the many European national movements. Inglehart and Woodward's article on language policy and conflict has strengthened the nature of the examples, especially in regard to the Austro-Hungarian Empire. Earl Rees' article on Spanish linguistic conflict in relation to the Catalan minority has also proved helpful in submitting Spain as a viable example of language policy used in schools to destroy a minority language.

France

As the first case-study, France provided an historical example of how linguistic policy utilized through the education system was successful in destroying minority language communities within France. Anne Stevens's book on French government provided the background information necessary to understanding the nature of diversity and France, and how that diversity has shaped the French experience. M.S. Anderson also includes information on France in his explanation of European nationalism. David Bell's article on language policy during the Revolutionary period gives a good understanding of how nationalism- and especially French nationalism- began. The variety of languages existing in France prior to the Revolution is also discussed, giving

⁸ Lowell W. Barrington, "'Nation' and 'Nationalism': The Misuse of Key Concepts in Political Science," *PS: Political Science and Politics*. 30 (Dec.,

Ukraine

Ukraine has been selected as the second case study, mainly due to the fact that it is still a “work in progress” in terms of the construction of a viable political and national entity. The bulk of research for this paper has focused on the Ukraine, mainly due to the fact that Ukraine speaks volumes regarding the future of nationalism and the different paths open to new nation-states who are recently independent from years of dominance. The future of nationalism lies with nation-states like the Ukraine, as they are not as yet complete projects in terms of structured political-cultural bodies.

Timothy Snyder’s book on the historical processes which have come to form modern Ukraine has proved helpful as a base of understanding the very complex nature of Ukrainian nationhood. Serhii Plokhy’s article on the myth of Ukrainian historiography offers a different perspective, as it exams Ukraine as a nation *without* a viable history, and therefore in need of creation myths to sustain the hope of future Ukrainian nationalism. Roman Szporluck’s article provides an assessment of education policy under Soviet control, detailing the two forms of educational ideology- nationalized language policies and Russified education systems- which Ukraine was subjected to while under the control of the USSR.

In regard to modern, independent Ukraine, Anna Fournier’s article details some of the problems facing Ukraine in terms of its large, vocal Russian minority. Janusz Tomiak provides an overview of modern education processes within the Ukraine, emphasizing efforts that are being made to increase Ukrainian nationalism through the renewed interest in the Ukrainian language. However, Petherbridge-Hernandez and Raby give a much different view of Ukrainian education. In their article they explore the problems facing Ukraine in terms of linguistic disinterest, and the proliferation of outside languages- English, German, and Russian- and how reliance on outside tongues will harm any attempts at solidifying Ukrainian nationhood.

Nationalism: Definitions

Since the establishment of the modern state system in the seventeenth century and the subsequent upheavals of the French Revolution

That nationalism as a force has had a powerful impact on western history (and, through the outreaches of colonialism, a similarly powerful impact on the rest of the world's history, to the point of providing models by which new states rearranged their political structures once the colonial powers had abated) is very difficult to deny. However, simply recognizing that a force *called nationalism* has altered the very core of our society, does provide the necessary level of information for understanding. Concepts are well and good in acting as buzz-words, as large ideas compressed into a handy label. However, this comprehension provides no answers, especially to those who have not spent their lives studying the concept. It is the aim of this section to provide a sufficient definition of nationalism as both an idea and an historical force. This definition in no way proposes to be the complete and final say on what nationalism means or encompasses. However, for the purposes of this study, a general understanding must be presented so that the subsequent arguments will have a firm foundation on which to stand.

Anthony Smith, one of the premier authors on the subject of nationalism tells us that

the term 'nationalism' is commonly used in three main ways. At the broadest level it refers to the whole process by which nations and nation-states have come into existence...A second meaning of the term 'nationalism' concerns of this broad set of processes, the formation of national consciousness and solidarity...The third usage is also the most specific, and it confines the term 'nationalism' to the phenomenon of an ideological movement.⁹

This definition gives an incredibly broad picture of all the things that could possibly be labeled as nationalism; all the political, cultural, and social movements and events that fall under the heading of what nationalism may represent. For the purposes of this paper, then, we shall eliminate two of Smith's methods of viewing nationalism, in order to streamline what, exactly, it is we are hoping to define.

presupposed to exist before any political structure- is a political event, wherein borders are drawn and territory is assigned in a political method. This process is more closely related to the idea of the state than it is to the idea of the nation, and, as we shall see, the nation and the state are very different entities.¹⁰ The third level of nationalism that Smith addresses deals in the strict terms of ideology, which, as we shall see, is an inappropriate assessment of nationalism within the context of this paper. Nationalism is not an ideology as much as it is a social structure which addresses not just a sense of political belonging but rather socio-cultural belonging.

So with that elimination, we are left with one of Smith's definitions, in which nationalism "concerns...this broad set of processes, the formation of national consciousness and solidarity." It is this set of processes which are the interest of this paper. They are divorced to an extent from the political processes of building a state, as these actions- forming national consciousness- do not require the presence of a state. Often, these processes come to light when there is the noticeable absence of a nation-state. More will be discussed on that condition in later sections.

Now that the varied things that nationalism can mean have been narrowed down, we can turn to a more definite characterization of what it is nationalism- in the sense of creation and consciousness- really denotes.

In defining nationalism, Lowell W. Barrington's article¹¹ has proved helpful for streamlining the varied characterizations available, particularly in light of the expansive literature available on the subject. While the definition of nationalism is a cause for conflict within the academic world, Barrington offers a useful guiding classification which fits the needs of this paper best. Barrington, however, reminds us that before we can discuss nationalism as a construct, we first must address the nation itself. Barrington first addresses some of the more common definitions before providing a streamlined version

¹⁰ A. Smith writes: "Very often this is designated the 'nation-building' process in the literature, although it is as much concerned with state-building

First and foremost, a nation is a collective of people. This is a necessary detail, but one that does not help us distinguish between nations and other groups in society. What makes nations unique is that they are collectives *united by shared cultural features (myths, values, etc.) and the belief in the right to territorial self-determination*. Thus, nations are groups of people linked by unifying traits and the desire to control a territory that is thought of as the group's national homeland.¹⁴ (Italics used in the original.)

In essence, the combination of an identifiable territory and its connection to a group who share noticeable traits- a collective history, and, perhaps most importantly, a collective language and linguistic tradition- is what shapes a

¹² Barrington, “”Nation” and “Nationalism””, 712. Barrington quotes Smith’s definition of a nation: “[a] named human population sharing an historic territory, common myths, and historical memories, a mass public culture, a common economy and common legal rights and duties for all members” (Smith 1991, 14). He also quotes Tamir, who finds fault with Smith, accusing him of providing no demarcation between: “[the] reasons for the emergence of a nation (a shared historic territory, a common economy, and a common legal system) with the results (sharing myths and historical memories).” (Tamir 1995, 424). Barrington himself, however, criticizes Tamir’s definition (“[a] community whose members share feelings of fraternity, substantial distinctiveness, and exclusivity, as well as beliefs in a common ancestry and continuous genealogy” (Tamir 1995, 425).) as being more appropriate to describing an ethnic group or culture, rather than a nation. Barrington takes issue with the absence of territory in Tamir’s definition, as Barrington sees territory as an asset when speaking about a nation. Barrington lauds Tamir as “an improvement over vague definitions that cannot distinguish between nations and classes...” however he reminds us that according to Tamir, “...it is unclear how a nation differs from an ethnic group using this definition” (712). Barrington continually stresses the importance of addressing

science.¹⁵ A state is a purely political entity; that is, “a state...is the principal political unit in the international political system corresponding to a territory, a relatively permanent population, and a set of ruling institutions.”¹⁶ In other words, a state can have within it many nations, and a nation is not a political body; however, states often coincide with a certain nation, or at least that has been the trend since the late nineteenth century.¹⁷

For the purposes of this paper, the term “nation” will be used because the focus is on nationalism. When speaking about nations, it must be understood that the nation is *not* the complete political structure. Ethnic Russians may be members of the Ukrainian *state*; however, they are not generally considered part of the Ukrainian *nation*.

In addition to Barrington’s definition, it is important for the purposes of this paper to explore briefly two other characterizations of nationalism. Both Ernest Gellner and Benedict Anderson offer their own definitions of what constitutes a nation and what it means to say that nationalism is a force both in terms of ideas and actions. The definitions provided by Gellner and Anderson will help reinforce their theoretical paradigms of nationalism, and it is essential to understand what exactly they mean when they utilize terms such as ‘nation’, ‘state’, and ‘nationalism.’

¹⁵ Barrington, ““Nation” and “Nationalism,”” 713

¹⁶ Barrington, ““Nation” and “Nationalism,”” 713

¹⁷ For example, the United States of America is a “state”, but within it are groups of Native Americans. These groups would be considered nations, as they each have their own territory, shared history, and language, although in essence, they are not political entities. Other examples can be taken from the great multinational empires of the nineteenth century, such as the Austro-Hungarian, Russian, and Ottoman Empires. Although these empires are also often referred to as *multiethnic* systems as well, Barrington’s definition of nationalism provides a firm argument for stating that these empires contained within them *nations*, as opposed to simply different ethnic groups. For example, under the rule of both the Russian and Soviet Empires, the Ukraine

saying: “Nationalism is primarily a political principle, which holds that the political and the national unit should be congruent.”¹⁸ This definition echoes Barrington’s characterization, as in order for the national unit to be congruent with the political unit, the territorial boundaries of the political unit must encompass the majority of a nation; that is, the boundaries must include the bulk of a nation’s population and be administered by an individual or group of individuals sharing the same nationality as the majority. For Gellner, this is the *principle* of nationalism, that the political and the national are harmonious¹⁹. If the political boundaries of a state, for example, separated a large portion of a nation, then this would stir a large level of nationalist sentiment.²⁰

For Gellner, “nationalism is a theory of political identity.”²¹ In this way, nationalism can be characterized as an ideology, but one both more complex and yet more basic than any other ideology, such as Marxism or

¹⁸ Ernest Gellner, *Nations and Nationalism* (New York: Cornell University Press, 1983), 1.

¹⁹ It is important to remember that nation and state are not the same. This concept cannot be stressed enough, as it sometimes seems confusing to say that a state must encompass a nation. It is a mistake made often, to say that nation and state are interchangeable. It is one the basic tenets of this paper to explain that nation and state are very different- and often conflicting- structures.

²⁰ Gellner on nationalist sentiment: “Nationalism as a sentiment, or as a movement, can best be defined in terms of this principle. Nationalist *sentiment* is the feeling of anger aroused by the violation of the principle, or the feeling of satisfaction aroused by its fulfillment. A nationalist *movement* is one actuated by a sentiment of this kind” (1). For Gellner, nationalist sentiment can be aroused by the violation of nationalist principals; for example, when the political boundaries of a state fail to include a majority of the members of a nation, or when the boundaries include other nations, or

Gellner, nationalism *presupposes* the existence of a political unit; that is, a state. As Gellner explains:

[I]t seems to be the case that nationalism emerges on in milieux in which the existence of the states is already very much taken for granted. The existence of politically centralized units, and of a moral-political climate in which such centralized units are taken for granted and are treated as normative, is a necessary though by no means a sufficient condition of nationalism.²³

The definition of the nation is a complex undertaking in Gellner's work. He makes it quite clear that, although history and nationalist sentiment dictate that the nation and the state seem to be destined to co-exist, and indeed, the state is the most effective tool of nationalism, the nation is not- and should not be- confused with the state. This is one point that is agreed upon by most scholars of nationalism.

Benedict Anderson also explores nationalism in terms of offering a definition on which to build his own theoretical framework. Anderson starts his process of defining by outlining the paradoxes of nationalism, and how these paradoxes make it very difficult to come up with a satisfactory understanding of how nationalism should be characterized.²⁴ Anderson works within the construct of these paradoxes in order to approach nationalism from

²² Anderson goes into detail about nationalism and its differences from other ideologies; this will be addressed shortly when Anderson's definition is explained.

²³ Ernest Gellner, *Nations and Nationalism* (1983), 4.

²⁴ Benedict Anderson, *Imagined Communities*, (London: Verso 1983), 5.

Anderson defines the three paradoxes of nationalism thusly: "(1) The objective modernity of nations to the historian's eye vs. their subjective antiquity in the eyes of nationalists. (2) The formal universality of nationality as a socio-cultural concept- in the modern world everyone can, should, will 'have' a nationality, as he or she 'has' a gender- vs. the irremediable

as much of a social construct as religion.²⁷ For Anderson, nationalism is the sentiment expressed in response to belonging to such an ‘imagined community’; that is, the shared qualities of a nation form the sentiment.

As with any other concept in the social sciences, there is a need among academics to categorize nationalism as something more than a phenomenon, that is, there is a need to give nationalism its validation, a reason for its appearance on the world scene and its continued influence as a viable force in international affairs. Ernest Gellner and Benedict Anderson both provide theoretical models for nationalism which have been of great use to political scientists, anthropologists and sociologists alike in writing about nationalism and its effects on different aspects of political and cultural life. The theories of Anderson and Gellner will be addressed in terms of their effects on education, as by themselves, the paradigms are expansive and rather esoteric in their focus. So with that in mind, we will now turn to education and nationalism as cooperative forces and the place of language

²⁵Benedict Anderson, *Imagined Communities* (1983). Anderson reminds us that “Part of the difficulty is that one tends unconsciously to hypostasize the existence of Nationalism-with-a-big-N (rather as one might Age-with-a-capital-A) and then to classify ‘it’ as *an* ideology. (Note that if everyone has an age, Age is merely an analytical expression.) It would, I think, make things easier if one treated it as if it belongs with ‘kinship’ and ‘religion’, rather than with ‘liberalism’ or ‘facism’.” (5) In other words, nationalism is not so much a socially constructed ideology along the lines of Marxism and the like, but rather a culturally pervasive state of being, in the way that we have kinship relations and religious identities. Nationalism is not, in Anderson’s mind, an academic exercise in categorizing a set of political or purely ideological beliefs. Rather, it is a state of being more primal, less developed.

²⁶ Benedict Anderson, *Imagined Communities* (1983), 6

²⁷ Benedict Anderson, *Imagined Communities* (1983)- in Ch. 1, Anderson explains that in his theoretical paradigm, one of the reasons for the rise of nationalism in the eighteenth century was the collapse of the ideas of

Nationalism, Language, and Education: Theory and History

Susanne Wiborg reminds us of Jean-Jacques Rousseau's prediction: that "not only could patriotism be created but indeed molded. The task of governments was to carve the mould- the education institution- in which the children of nations should be shaped."²⁸ As early as the Enlightenment, Rousseau and others²⁹ saw the value of "national education systems"³⁰ in promoting and strengthening the existence of a national identity. Rousseau's political nationalism was best furthered by education, as was Johann Herder's more cultural brand of nationalism. Both aspects of nationalism are addressed in this paper- the progression of nationalism in France was most definitely an exercise in political advancement- both of a national community (the French) and a political entity (France). In contrast, the condition of Ukrainian nationalism is much more cultural- more along the lines of Herder's conception of nationalism. Because of the fact that the Ukraine spent many years under outside political dominance, Ukrainian nationalism had to develop in a much less structural form. However, despite the different courses nationalism has taken in these cases, one factor is consistent: the presence of education as a conduit for passing on nationalist sentiment to the younger generations of a community.

Because nationalism relies upon a community wherein the members share basic features- a similar concept of history and myth, and most importantly a common language- a structure is necessary for disseminating the connective cultural knowledge. The structure which has proved (and continues to prove) most effective in spreading nationalism within Western

²⁸ Susanne Wiborg, "Political and Cultural Nationalism in Education. The ideas of Rousseau and Herder concerning national education." *Comparative Education*. 36 (May, 2000): 235-243, 1. The word *patriotism* is used in this statement, because in the French context, nationalism was much more centered on the political (i.e. the state), rather than a more cultural, Germanic version of nationalism. This distinction will be discussed presently.

nation. Both Anderson and Gellner stress the education system in their theoretical approaches to nationalism, and how education is one of the basic tools for creating-whether intentionally or unintentionally- a national identity. No other system is so far-reaching and intertwined within a community as the education system. In terms of the Western experience, it is safe to say that almost every child is exposed to the education system of their respective communities.

Wiborg tells us that “the notion of education as being a useful tool for developing a common national identity within specific geographical borders arose in the late eighteenth century.”³² This time period coincides with Anderson’s observation of nationalism’s birth as a construct (in other words, the beginning of an imagined community formed as ‘nations’).³³ Both authors identify the importance of the breakdown of religious universalism in the formation and strengthening of nationalism. Wiborg traces the use of the school before the eighteenth century as “primarily...a vehicle for fulfilling an ecclesiastical need.”³⁴ However, the eighteenth century saw the disintegration of religious universalism; although religion still was to play a large part in European politics, the cosmopolitan desires for a supra-national entity based solely upon shared spirituality were quickly being replaced by a fractioning of different cultures and groups determined to have their own separate

³¹ It is probably quite apparent that the bulk of this paper focuses on the European experience in specific and the Western experience in general in regard to nationalism and nations. This focus is due to the fact that nationalism is in essence a Western concept. The idea of the nation and therefore of nationalism as a sentiment came to fruition in Europe, and was spread across the globe by colonialism. So while there are hundreds of thousands of nations all over the world, the *idea* of the nation was born in Europe during the eighteenth and nineteenth centuries. At heart, nationalism is a Western construct.

³² Wiborg, “Political and Cultural Nationalism in Education.” 236

Although the first faint stirrings of nationalism can be traced quite far back in Europe's history, it did take quite a few decades to gather its strength. It is not until the mid nineteenth century that nationalism emerged as a powerful force in matters of state and society. As nationalism was coming into its own and education was beginning to reflect this rise, the ideas of two European thinkers began to define the different paths nationalism and education were taking. It is important to gain a basic understanding of Rousseau and Herder's brands of nationalism in the context of education, as these two styles will be important in tracing the growth of nationalism as a viable force in both ideological discourse and public affairs. Susanne Wiborg describes Rousseau (a Frenchman) as a political nationalist, while nationalism according to Herder (a German) took on a more cultural character.³⁷ These two divergent ideas of nationalism have been particularly interesting in relation to the case studies selected, as described earlier.

M.C. Anderson identifies Rousseau as one of the first to understand the importance of nationalism in establishing modern governments. According to Anderson, "far more than any other writer, Rousseau begins the close association of nationalism with democracy, which was often important in the nineteenth century."³⁸ It was precisely this parallel between nationalism and democracy which cast Rousseau's conception of the nation and its accompanying sentiment in a political light.

Rousseau's political nationalism is basically an external process. Belonging to a nation (or, in a simpler sense, belonging to a specific community, an identity group with the same basic history and culture) in Rousseau's reasoning, is an intrinsic quality of all people; however, this intrinsic possession is not the end-all of national sentiment. Rather, nationalism is created by a government- by a state-system in order to reinforce that system. This reasoning is based on the assumption that the

³⁵ Although religion lost its major claims to universalism, spirituality did eventually become a facet of nationalism, and is often lumped in with other national identifiers such as language and history.

is based on a deeper level of belonging, which *comes before* the idea of the state. States are manufactured creations built upon an intrinsic connection of deeper cultural relevance.³⁹

So what does this have to do with education? For Rousseau, the state is responsible for building the nation, just as the nation is responsible for sustaining the state. Wiborg tells us that: “Rousseau believed that the people are what their governments make them. He saw the institutions of the state as instruments by which the national character can be fashioned.”⁴⁰ The school system is one of the most (if not the most) powerful of all state institutions. It is the institution which affects the greatest number of individuals within a nation-state, and it is the institution which is *the most uniform in its effect*. In other words, and this is where Gellner’s High Culture Theory can be applied, a successful nation-state, wherein the national quality and sentiment of the major cultural group are stressed through the political actions of the state-structure, there will *always* be a uniformity in what is being taught to the future of the nation-state’s population. In Rousseau’s mind, education for strengthening nationalist sentiment is a top-down process, whereby the state structure recognizes its ability to determine what exactly defines its citizens, and then sets up a system to constantly reinforce that citizenship.

This process is inherently political. It is born of a structure set up for political ends, rather than drawn out of any cultural wellspring. Rousseau’s nationalism reflects his ideological origins as a proponent of contractarian theory and is born of the Enlightenment’s call for reason and rationality. Rousseau views nationalism through the prism of the French experience, and hence, Rousseau’s opinion on education’s importance for nationalism’s

³⁹ Wiborg, “Political and Cultural Nationalism in Education.” 236-237.

Wiborg’s words: “The object of Rousseau’s works is not to ‘discover’ what a nation is, but to illustrate how a body of individuals is formed into a nation-state, that is, how rightful political authority is born...however, there can be no state without common cultural similarities on which to build. Otherwise culture is merely an external fact and not the outward expression of a person’s inner being. To experience a political culture from within, people

Johann Herder formed his idea of what a nation and nationalism had to do with education in a very different atmosphere than Rousseau. Herder's Germany⁴² was in the first stages of the Romantic Movement, a socio-cultural movement which would transform Europe from a bastion of rationality to a more culturally-driven conglomeration. Herder read the Enlightenment literature concerning the nation and Rousseau's pieces on education, and although these works influenced his writings, he eventually came to much different conclusions concerning education and nationalism.⁴³

Whereas Rousseau's nationalism was a rather contrived force, brought about by the political forces governing a nation-state, Herder saw nationalism as an organic movement which evolved- rather than existed- as the product of manufactured circumstances.⁴⁴ The state did not have the ability to fashion citizens- to be a citizen of a nation was an inherent quality that evolved over time. Nations were not subject to the formatting of political structures; they, instead, were living, breathing entities of their own right,

⁴¹ Wiborg, "Political and Cultural Nationalism in Education." 238. Wiborg notes that although Rousseau was a product of the Enlightenment, and his ideals were formed using the Enlightenment's general tenets, he differed in his rejection of cosmopolitanism as the pathway to eventual freedom. Rousseau, instead, saw the nation-state as a more appropriate system, and this connects him to Herder. Although the two men had very different ideas, there is a definite progression from Rousseau to Herder in terms of the place of the nation and nationalism in European intellectual discourse.

⁴² By Germany, it is understood to mean the collection of German States that would eventually come together to form modern Germany. Germany was not a unified nation during Herder's time, although the idea of Germany was an important concept for the cultural nationalists.

⁴³ Wiborg, "Political and Cultural Nationalism in Education." 239.

⁴⁴ Wiborg, "Political and Cultural Nationalism in Education." 239. Wiborg goes on to explain that, "Both [Rousseau and Herder] held the view that society should be an expression of nature; however, their understanding of nature was different. For Rousseau the nature stage had unfortunately

of the Enlightenment. However, for the purposes of this paper, the most important aspect of Herder's cultural nationalism is his focus on *language*.

As will be delved into in much greater detail in later sections of this paper, language is arguably the most important aspect of both cultural and political nationalism. Both Rousseau and Herder emphasize language, although Herder's concept of the mother tongue is emphasized in a quite different way. Still, political nationalism relies on language as well, for a uniform language is necessary for a community to form the cohesive bonds essential for survival and strengthening. In Herder's writings, however, the idea of language as a political force gained prominence, a change which is still affecting struggles for national sovereignty all over the world. Herder's focus on language as a form of both cultural *and* political identity changed the nature of what it meant to belong to a nation. As Wiborg states, "a nation no longer simply meant a group of citizens united under a common political sovereign...rather it was to be regarded as a separate natural entity whose claim to political recognition rested on the possession of a common culture rooted in language" (emphasis added).⁴⁵ In other words, nations become divorced from any convenient political reality- they are not created by a structure, but rather warrant a structure of their own creation⁴⁶. What this means is the nation itself is the base of all political reality. And all political claims the nation possesses are derived from the idea of a shared language. Language is the foundation on which all conceptions of the nation and nationalism are based, for without a shared language, communities could not express any sense of related past or shared experience. The presence of a mother tongue and its attached importance create a cohesion that is essentially tangible and instantly recognizable. The specters of history and tradition are only as real as the words that bind them to significance.

These two conceptions of nationalism would grow to define the European social and cultural structure from the mid-nineteenth century onward, especially in terms of language policy and education. The schools were the appropriate bastion of nationalism, as the schools were and continue to exist as the birthplace for standardization. The schools are primarily

nationalisms of Rousseau and Herder in mind, and attention to Gellner's Theory of High Culture, a brief historical overview of varied national movements and their place in the schools (especially in terms of linguistic policy) will be presented. This historical survey will provide a few references to which the case studies (France and Ukraine) can be compared, and also provide opportunities to encourage more research into the detailed educational and linguistic policies' interplay within the nation-states described.

M. S. Anderson, in his volume on nineteenth century Europe, traces the growth of national education systems, equating that growth with the spread of nationalism as a viable sentiment and motivation. Anderson notes that it was imperative for the school system (along with other media outlets, such as newspapers) to create a broadly similar experience for all students within a nation in order to form a level of cohesiveness which would replace the smaller communities of villages and other separate identity groups.⁴⁷ After all, the nineteenth century's primary concern in Europe was the alignment of the national with the political, which meant both shrinking the borders of large multinational empires and also bringing similar, smaller cultural groups together to form more powerful and effectively competitive nation-states.⁴⁸

The idea of the school as the bastion of nationalist sentiment began in the early nineteenth century, and the process of creating national schools

⁴⁷ M.S. Anderson, *The Ascendancy of Europe: 1815-1914*, (Harlow, UK: Pearson Education Limited 1985.), 215. Anderson explains: "There were two media in particular through which linguistic nationalism could be fostered. These were the schools and the press, above all the newspaper press. Both of these powerfully fostered the growth of cultures which were not local and traditional, not based on subgroupings of a relatively small and even intimate kind such as village or regional communities, but increasingly homogenous over large areas, areas defined by state frontiers. *These new national cultures were based on mass access to the printed word and the ideas it conveyed*"

...in Prussia a department of public instruction was established in 1807...and the abolition of fees in state schools was beginning in 1833. In the same year France began to create a system of national elementary schools. Holland began the creation of a national system of education in 1806, Greece in 1823, Belgium in 1842, Portugal in 1844, Romania in 1859, Hungary in 1868, and even Bulgaria, still not formally an independent state, in 1881. The adoption of a uniform national curriculum in German schools in 1872 was one of the most important efforts to consolidate the political unification of 1866-71.⁵⁰

What is most important about the establishment of these national education systems is the language that was being taught. Nationalism required standardization; a nation could only call itself a nation if its members could communicate with one another (even if the community members did not actively communicate with other community members from different areas of the nation, the very idea that they *could*, that they shared a language with an untold number of unknowns was enough to give the nation weight in the imagination) in a language which would symbolize the significance of being a part of the nation just as its shared history and myths symbolized membership.

⁴⁹ M.S. Anderson, *The Ascendancy of Europe: 1815-1914* (1985), 215-216.

According to Anderson, German philosopher Fichte saw the school system as a method for the “recreation” of Germany as a cohesive nation, rather than a conglomeration of smaller states. Another German nationalist, F.L. Jahn, (described by Anderson as “the most xenophobic of the German nationalists of the period”, 216) “had drawn up a detailed programme [sic] of national education which stressed patriotic history, compulsory manual work, gymnastics and military sports.” 216.

⁵⁰ M.S. Anderson, *The Ascendancy of Europe: 1815-1914* (1985), 216.

Anderson goes on to describe the creation of national education systems in Europe: “In the more backward parts of Europe the growth of primary

that the true power of the schools as the main apparatus of the dissemination of nationalism came to the forefront. Gellner reminds us that it would be impossible for every nation to have its own sovereign state structure; there are simply too many nationalities for each to be an independent entity.⁵¹ Therefore, many states (especially in Europe) contain a variety of nations, especially in terms of language communities. Of course the great multinational empires of the nineteenth century contained a multitude of nationalities: Austria-Hungary, Russia, and the Ottoman Empire all ruled over an incredibly diverse group of subjects, with many different language communities. The Austro-Hungarian Empire will be discussed in some detail in the following section.

However, even the established states of Western Europe found a level of conflict in creating a cohesive structure to encompass the majority nationality. Spain, France, and Italy all had issues of linguistic dominance in their formative years as struggling nation-states, as they attempted to reconcile the national and the political in a way that would give the most power to the dominant national identity group (and precedence to that group's language). France is one of the two case studies selected, and hence will be given a great deal of attention later on. For now, however, the Spanish struggle provides a telling illustration regarding the position and importance of language in the national-political struggle for dominance.

Like most European countries, Spain was once a structure that contained many different language groups. Before the modern age, these language groups were entities unto themselves, as communication over long distances was difficult and time consuming. Specific communities (sometimes as small as villages) were self-sustaining units where language developed according to local custom and culture. Inglehart and Woodward explain that:

⁵¹ Gellner describes this caveat in his definition of nations and nationalism. Because (as quoted earlier) Gellner defines the fulfillment of nationalist sentiment as the congruence of political (i.e. the state) boundaries with the

of social and economic mobility [such was the state of most European communities before the eighteenth century] the problem [of many languages] is largely irrelevant- even if such populations have a linguistically-distinct elite group.⁵²

However, as modernity reformed the nature of communication and the desire for a consolidated state consumed Spanish leaders, the issue of language became important in terms of power and control.⁵³

Earl Rees' article on Spanish linguistic normalization in the Catalonia region illustrates how contentious the issue of language truly can become in the face of national dominance. As Rees explains, "for Spanish nationhood, Catalonia has posed a centuries-long source of controversy reflecting that region's separatist sentiments and sometimes strenuous efforts to preserve its distinctive language and culture."⁵⁴ Although the Catalan language is far from the only distinctive language in Spain, it is one of the most dangerous because of its level of standardization. The Basque languages, in contrast, are hardly standardized in terms of differing dialects and written accounts. So while Basque separatists are just as adamant in their desire for independence as Catalan separatists, the fact that Catalan is a standardized language provides that nationality with an added edge- and also portrays them as a very real danger to Spanish nationhood in the way the dominant language culture of Spain would like the reality of Spanish nationhood to function.⁵⁵

⁵² Ronald F. Inglehart & Margaret Woodward, "Language Conflicts and Political Community." *Comparative Studies in Society and History*. 10 (Oct, 1967): 27-45, 28.

⁵³ Inglehart & Woodward trace this progression in general terms: "when the general population of a society [in this case, Spain] is going through the early stages of social mobilization, language group conflicts seem particularly likely to occur; they may develop animosities which take on a life of their own and persist beyond the situation which gave rise to them" 28.

⁵⁴ Earl Rees, "Spain's Linguistic Normalization Laws: The Catalan

the school system, and whether Catalan could be offered in schools. For after all, if a language is not taught, then it will not persist. Languages die out when there are no new speakers being produced, and when languages die, the cultures that spawned them die as well.

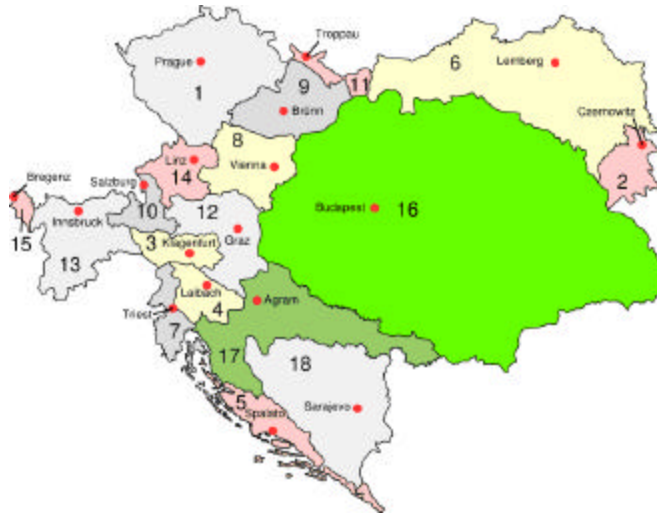
The issue of linguistic domination and extinction, and the problem of which language should be taught in schools were both common concerns in Europe during the nineteenth century, and persist to this day. Conflicts have arisen in Northern Italy between those who speak Serbian and the dominant Italian language. Great Britain has seen conflicts in regard to Gaelic as a viable language in Scotland, and the Welsh language. In many of the Soviet successor states (the Ukraine included), there are continued conflicts between those who speak Russian and those who wish to develop the national language. Stepping away from Europe for a moment, language conflict is occurring in Canada, as separatists in Quebec demand French as their national language, while immigrant communities in the United States wish to keep their own national language customs alive.⁵⁶

However, according to Woodward and Inglehart, “one of the most important cases of language conflict in modern times has been the Austro-

standardized, full-fledged Romance language with a structural development comparable to Castilian Spanish.” Rees makes the distinction between Catalan and Basque: “On the other hand, Basque and Gallego- Spain’s other significant minority languages- have not generated analogous bodies of literature [analogous to what has been produced within the Catalan language community] and their linguistic development lags behind Catalan.”

⁵⁶ Very recently, the question of Spanish as a second language in the United States has become a volatile issue. A rendition of the ‘Star Spangled Banner’ being sung in Spanish has raised the ire of many purists in America, as has the idea of Spanish being taught *in the schools* as a second language; that is, regular classes being offered in Spanish for the large minority of Hispanic immigrants. While the issue of language in the United States is quite different

that would become part of Romania, Czechoslovakia, and Poland:



(Austro-Hungarian Empire, divided into sections based on nationality. *map taken from: http://en.wikipedia.org/wiki/Image:Austria-Hungary_map.svg, subject of public domain*)

The ‘official’ language of the Empire (up until 1867)⁵⁸ was German, as German was the language of Vienna, the Empire’s capital. During the early part of the nineteenth century, before the crown of the Empire was divided, attempts were made to squash the efforts of minority language communities. Emperor Joseph II “attempted to establish German as the universal language of the Empire,”⁵⁹ and with this mandate, the movements

⁵⁷ Inglehart & Woodward, “Language Conflicts and Political Community.” (1967), 34.

to represent the basic identity of a community. Herder's writings on the mother tongue and its cultural significance to a people permeated all levels of European society, especially within the great empires that dominated many language groups. This battle for linguistic freedom took place in many fields, but none so important as the school. For as educational reform swept through Europe- even through the backwaters of the Empire- it became obvious that education had the power to transcend problems of distance and poor communication between regions. If all children of a language group- of a nation- were being taught the same things, then it did not matter if members of the community never came into contact with one another. A sense of connection was instilled in the knowledge that students in all areas of the nation were learning the same subjects, the same language, and that if the opportunity arose, any member of the identity group could communicate with any other member based on their shared knowledge of the same language. This shared knowledge also translated to shared power. For if all members of an identity group were taught the glory of their nation and the strength of their connection, it would prove quite a difficult task governing them, especially if the government was distant, and foreign. As language policy became a pressing issue, it would prove exceptionally difficult for Vienna (and after 1867 Vienna and Budapest) to govern far-off lands such as Transylvania and Galicia.

After 1867, when the Austrians ceded half of the Empire's crown to the Hungarians, German and Magyar (the language of Hungary) shared dual significance.⁶⁰ However, hundreds of other languages and dialects were

⁶⁰ Inglehart & Woodward, "Language Conflicts and Political Community." (1967), 35. After Austria was defeated by Prussia in 1866, the Habsburg monarchy was forced to reform the Empire's political structure. The Hungarians were the second-largest group within the Empire, and their increasing power and influence allowed them to take advantage of the weakened Habsburgs in pushing their demands for dual leadership. This movement culminated in the *Ausgleich*, wherein "the empire was divided into two parts and re-labeled the Austro-Hungarian Monarchy" (35). Inglehart &

policy illustrates the essential importance of national languages. The eventual dissolution of Austro-Hungary after the First World War was a brief reprieve for the nations which had suffered under German/Magyar dominance-however the expansion of the Soviet Union soon engulfed the nations of Eastern Europe, bringing on decades more of dominance from a far-away capital, with a foreign language.

The examples of Spain, Italy, and Austro-Hungary are only a few of the many cases of linguistic conflict and its place in the schools. The overarching concept, however, is *power*. Language is a powerful force; in fact, it may be one of the most powerful political forces. Nationalism would not be the prevailing and compellingly dominant movement that it has become without the presence of a national language. Of course, this may all sound rather redundant, but it is important to give language its due when speaking about nationalism. Language is almost always one of the top priorities of nationalists, when they demand sovereignty. As has been shown above, nationalists in Spain, Italy, Canada, the Austro-Hungarian Empire, and even- to an extent- in the United States preach first and foremost that their respective languages are just as relevant and necessary as the language of the political structure in which they are located.

The next section of this paper will examine two more examples of nationalism's place in education policy in terms of language usage and control in much greater detail. First, France will be discussed as the prime example of schools used to both promote a national language and destroy secondary languages within the political structure. Next, the Ukraine will be brought into focus as a nation that- until recently- has been the victim of control by an outside force. Where the Ukraine is headed in terms of nationalism and language within its schools will be discussed. Both Ernest Gellner and Benedict Anderson's theories will be invoked, along with parallels to Rousseau and Herder.

Case Study: France

The idea of France as a nation seems as impenetrable and constant as a monolith. The deference paid to the mother-tongue in France today

unerring sense of 'being French.'⁶¹ However, the truth is much different. France has not always been a united nation, nor have the French even been 'the French' in the way we understand that characterization to mean in modern times. The French as a whole are a recent creation, who can trace their birth to the French Revolution of 1789, where France as a nation was fully conceptualized and set about on the road to creation and strengthening. One of the most important aspects of this creation was the standardization of a single French language.

'French' as a language is among the most recognized of the European tongues. During the Enlightenment, most of the European aristocracy spoke French, and the language was considered the speech of choice for the cosmopolitan-minded European upper class; France came to replace Latin as the language of intellectual and artistic discourse during the eighteenth and nineteenth centuries in Europe, and leaders from Catharine of Russia to Frederick of Prussia spoke French in their courts and even personal correspondence. Ironically enough, however, French was not the dominant language in France- even though this may sound counterintuitive, France struggled with language issues for much of eighteenth and nineteenth century. In fact, according to Stevens, as late as 1863, up to 25 per cent of the French population did not speak French, but rather one of the many other languages within the political structure of France.⁶²

France was once an entity of many languages. While French was spoken in Paris and the surrounding areas, the rest of France was divided into the Occitan languages of the South (the Languedoc region)- which included Gascon and Provençal- the more Teutonic languages of the northern periphery, which included Breton, Flemish, and German. Around the Western and Eastern borders, languages from Spain (Basque and Catalan)

⁶¹ Anne Stevens, *Government and Politics of France* (Palgrave Macmillan; 3rd. Ed., 2003) 1. Stevens writes: "the French constitutions of 1946 and 1958 proclaim France as a secular, democratic, social but also indivisible republic. Like many constitutional pronouncements this is a statement of will and

nor were they half-formed peasant tongues (although the French-speaking dominant culture attempted to portray them as such). Just as Catalan in Spain is a valid language with its own rules and advanced development, the varied tongues in France were all valid languages in their own right. These languages were taught to children; literature was written and records were kept in these tongues.

In general, the collective body of non-French languages present in France were known as *patois*, a rather derogatory term that, according to Bell suggested “a half-formed, earthy, unsophisticated language, caught part way between animal grunts and true human speech, a language of people not suited for political participation.”⁶⁴ With the advent of the French Revolution and the first stirrings of French nationalism- using ‘French’ as a collective term- the desire for a nation of one language became a pressing political concern for the architects of post-revolutionary France.⁶⁵ The Jacobin leaders made the first attempt to destroy the *patois*, beginning with a campaign in 1794 “to eradicate *patois* entirely and make the French language uniform throughout the Republic.”⁶⁶

This structural attention given to the issue of language and its potential for influence and power gives a clear illustration of the beginning of modern political nationalism, and what this nationalism would entail. Harnessing Rousseau’s ideas regarding the state’s ability to fashion the character of its attached nation, the French leaders began to “draw the common modern equation between the legal category of nationality and the cultural fact of language.”⁶⁷ Granted, this is not the cultural nationalism that most are familiar with. There was no universalism attached in that all French

⁶³ David A. Bell, “Lingua Populi, Lingua Dei: Language, Religion, and the Origins of French Revolutionary Nationalism.” *The American Historical Review*, 100 (Dec., 1995): 1403-1437. 1405.

⁶⁴ David A. Bell “Lingua Populi, Lingua Dei.” 1405.

⁶⁵ David A. Bell “Lingua Populi, Lingua Dei.” 1405. Bell explains; “France’s multiplicity of languages became a political issue as early as January 1790.”

⁶⁶ David A. Bell “Lingua Populi, Lingua Dei.” 1405. Bell quotes Jacobin

German states or other surrounding political entities. There was no overall desire to create a *universal* French nationality, but instead to strengthen and maintain *France* as it was, and increase the power of France along political lines, rather than cultural aspects. This, according to Bell, differentiates French nationalism from Herder's cultural German nationalism, and the nationalism that would rise in Eastern Europe.⁶⁸

So where does this place education? The simple answer is, education was the vehicle of standardization in France. The Revolutionary government began the task, assigning the education system the responsibility of eradicating regional languages and instilling French as the national tongue, a process which came to fruition during the Third Republic.⁶⁹ Although it took most of the nineteenth century to standardize French as a language, the task was accomplished. Many of the regional languages in France have been decimated to the brink of utter destruction, while others have simply been completely destroyed. The French education system made it quite clear that in order to function successfully in France- that is, in order to lead a profitable existence, in order to take part in the market economy- an individual had to be able to speak the national language. The education system of France accomplished its goal because it made the French language a *necessary commodity*. The notion of language as a commodity can be found in Ernest Gellner's High Culture paradigm, which can be applied to the task of the French education system in standardizing the linguistic practices in France.

Gellner's High Culture paradigm relies on the:

general imposition of a high culture on a society, where previously low cultures had taken up the lives of the majority, and in some cases the totality, of the population. *It means the general*

⁶⁸ David A. Bell "Lingua Populi, Lingua Dei." 1405.

⁶⁹ David A. Bell "Lingua Populi, Lingua Dei." 1406. The Revolutionaries were the first to charge the education system with the task of creating a

Establishment of an anonymous impersonal society, with mutually substitutable atomized individuals, held together above all by a shared culture of this kind, in place of the previous complex structure of local groups, sustained by folk cultures reproduced locally and idiosyncratically by the micro-groups themselves.⁷⁰ (italics added)

In the case of France, the 'high culture' represents the French-speaking, Paris-based leadership, which imposed linguistic dominance over the 'low cultures' of the *patois*-speaking individuals who made up the rest of the French population. Now, this paradigm offers an interesting paradox. It is obviously a tool of nationalism. The creation of French nationalism during the Revolutionary period demanded a process of disseminating this new nationalism over a highly diverse population. However, the imposition of a high culture over a wealth of 'lower' cultures also goes a long way to *destroying* nationalism in the way that nationalism is understood- that is, the sentiment expressed by an identity-group in regard to territory and governance. The nationalist sentiment of those in the Languedoc region of France was quashed by the greater French nationalism employed by Paris' leadership.

As Gellner points out, the driving force behind the imposition of a high culture is the school system. O'Leary, in his assessment of Gellner's writings, points out that "Literacy, an essential aspect of a normally socialized human being, is generated by state-sponsored educational systems."⁷¹ Education is the factory which produces nationalists because it is the manufacture of a national linguistic consciousness. As stated before, language *is* a commodity- it is a valuable possession that can determine a person's social stature and economic gain. In France, the language of the market became French. If an individual did not speak French, that individual could not reasonably be expected to participate in the market, and therefore their chance for economic gain were severely crippled. Language became the tool of nationalists (especially *political* nationalists, who saw the state as having the ability to fashion nationalist sentiment just as it fashioned political

of this culture, one must acclimate or be relegated to the sidelines of social, cultural, political, and economic participation.

Industrialization and modernization assisted the process. Gone were the individualized existences of farmers and other agricultural tasks, where communication with the outside world was limited, and a national language really unnecessary. Industrialization required, as Gellner recognized, a rather interchangeable population, where everyone spoke the same language, and therefore could be used as a viable workforce, without the challenge of communication in different tongues. Industrialization blew open the job market, sparked the movement of people from the rural areas to urban centers, and brought people in contact who- before the industrial period- would never have come together. In this atmosphere, the education system was even more important, as it trained future workers in the language of production. In France, a powerful industrial nation-state in the nineteenth century, the variety of languages was a detriment to industrial improvement.

It can be argued that the French example of education's role in promoting nationalism echoes directly Rousseau's ideas of political nationalism and Gellner's theory of High Culture. Overall, the French experience has a definite air of social and economic pragmatism; that is, while a nationalist sentiment in the more cultural did definitely exist, it was backed by a firm grounding in political and economic realities. France was determined to become a *nation-state*, a center of power and influence stemming from the solidarity of a people unified beneath a single language.

France serves as a historical example of how nationalism envelops the school system, turning education into a factory for distributing the commodity of language to subjects of a consolidated nation-state. We will now turn to a more recent example of nationalism and education: the Ukraine. The situation in Ukraine, as we shall see, is much different from what it had been in France during the period of French nationalization.



(Ukraine in relation to Europe. *Map taken from:*
http://en.wikipedia.org/wiki/Image:Austria-Hungary_map.svg, *subject of public domain*)

According to a study conducted by the University of Maryland and the Moscow State University, the separation of the Ukraine from the USSR in 1991 following the collapse of the Soviet Empire was unexpected and bitter. As the study explains, if “under Soviet socialism, Russians were ‘the first among equals,’ Ukrainians were the second.”⁷² This sentiment implies a very close relationship between Russians and Ukrainians, a view which many Russians presently still adhere to, as can be seen in Russia’s recent involvement with Ukrainian politics. However, the truth of the matter is much more complex. Ukraine is presently a nation-state on the crossroads of history. Ukrainian nationalism is at a point where policy decisions- especially in terms of education- will have a huge impact on the future of what it means to be Ukrainian.

The history of Ukraine as both a nation and a state is deeply tied to Russian history- in fact, the first Russian civilization was born in Kiev, the capital of Ukraine. The history of Ukraine is also a history of possession and control by outside forces. From the early modern period (mid 1500s) to the collapse of the Soviet Union in 1991, Ukraine belonged to the various incarnations of Lithuania, Poland, the Muscovite leadership (which became the Russian Empire), the Austrian Empire, the Russian Empire, and finally

and as a political entity in its own right⁷⁴. However, a complete historical analysis of Ukraine is beyond the scope of this paper, and as such, we will concentrate on recent Ukrainian history in terms of the direction Ukrainian nationalism seems to be taking, especially in regard to that sentiment's place within the Ukrainian educational system.

Education in Ukraine during Soviet domination followed two different models. The first model reflected Lenin's ideology regarding education in the Soviet Republics. According to Harold Weinstein (as quoted in Roman Szporluck's article), Lenin believed that literacy was the first step to consolidation; that is, literacy would be the binding factor which would aid the dissemination of Communist ideology. Lenin saw education and the education system as the tool of spreading Communism through its broad base. For it is important to remember that Communism is an ideology of *universalism*; in other words, the purpose of Communism is to eventually spread throughout the world as a unifying force between individuals from all national backgrounds. Communism acted as the antithesis to nationalism, in

⁷³ Timothy Snyder, *The Reconstruction of Nations: Poland, Ukraine, Lithuania, Belarus 1569-1999*. (New Haven, CT: Yale University Press, 2003) 105-132. The history of Ukraine in terms of linguistic development is complex. Snyder traces the progression of linguistic dominance from Orthodox-inspired Slavonic tongues to the acceptance of Polish as the dominant tongue during the Renaissance period. After Russia gained control of the territory, the Russian tongue came into power as well.

⁷⁴ See Serhii M. Ploky's article "The History of a "Non-Historical" Nation: Notes on the Nature and Current Problems of Ukrainian Historiography" *Slavic Review*, 3 (Autumn, 1995): 709-716. Ploky explores the historical 'myths' of the Ukraine, in the face of Ukraine's noticeable absent sense of actualized historiography. Ploky calls Ukraine one of the "so-called non-historical nations of Eastern Europe, whose nationalism and nation-states (much more than those of the west European countries) can be viewed as direct products of a highly elaborated historical myth" (709-710). Ploky identifies two of the main myths of Ukrainian nationhood: "The first [is]

Communism would be taught in could not necessarily be Russian. Although Russia was the ‘first among equals’ under the Soviet system,⁷⁵ the political reality of the USSR was that it encompassed hundreds of ethno-national groups, speaking hundreds of different languages and dialects. At its height, the Soviet Empire stretched from Eastern Europe to Central Asia, and of course, each of these areas possessed their own native tongues. Thus, Lenin understood that it would be impossible to encourage the propagation of the Russian language in non-Russian territories if the populace could not even read or write in their native languages. Since literacy was the predetermined goal of Lenin’s education system, the Soviets came at schooling throughout the Empire with a fourfold approach:

1. The schools would embrace all children. In other words, the only way Communist ideology could be disseminated throughout the expansive and varied population of the USSR was if all children were able to attend school and receive the message of indoctrination.
2. The schools would promote Communism. This is a rather straight-forward goal.
3. The schools would nationalize by introducing the language of the population. So, Ukrainian schools would teach in the native language of Ukraine, just as Polish schools would teach in Polish, and so forth.
4. Finally, the schools would encourage Russian as a second language, in hopes that once literacy was achieved in the native language, the children would then be exposed to Russian- the ultimate mother tongue of the Soviet system.⁷⁶

⁷⁵ In other words, although under the Soviet system- and the Communist system in general- all nationalities/ethnicities were supposedly ‘equal,’ due to the fact that nationalism could not exist as a force under Communist ideology. However, the political reality of the Soviet state was that Russians dominated the system. The USSR was, in essence, the latest reincarnation of the Russian Empire, under a different name and with supposedly different

language. The schools were allowed to teach students in their national tongue, and where language is dominant, culture cannot be far behind. The connection between language and culture is as old as the French Revolution, and the understanding that the possession and- more importantly- the *legitimization* of a national language in the face of outside dominance is a silent acceptance of that language's power.⁷⁷

In order to counter rising nationalist sentiment, the USSR began a different course in education.⁷⁸ Russian was to be the language of both social and economic interaction. Russian was to become the 'high culture,' to be imposed upon the lower culture of the satellite republics.⁷⁹ Much in the same way French became the language of the market in nineteenth century France, Russian became the language of the market in the Soviet Republics. Identity groups were forced to assimilate to the Russian tongue, although this came with varying degrees of success. Szporluck points out that assimilation in Western Ukraine of Russian as the mother tongue was slow in coming. According to Szporluck, in comparison with Belorussia⁸⁰, "the first postwar Soviet population census, held in 1959, revealed that Western Belorussians were more assimilated to the Russian language than were west Ukrainians. By 1970, this differential had widened."⁸¹ Szporluck posits that one of the major factors in Ukraine's slow assimilation of Russian was due to the fact that many Ukrainian newspapers were still written in the native tongue, allowing for a continued interest in and usage of Ukrainian as a viable language.⁸²

This system of assimilation remained in place until the break-up of the USSR in 1991. With the dissolution of the last great manifestation of a great Russian Empire, Ukraine was finally allowed its place as a free nation-

Tradition, Social Communication, and Linguistic Assimilation," *Soviet Studies*, 31 (Jan, 1979): 76-89.

⁷⁷ Roman Szporluck, "West Ukraine and West Belorussia" 76-89.

⁷⁸ Roman Szporluck, "West Ukraine and West Belorussia" 76-89.

⁷⁹ See explanation of Gellner's High Culture theory in the section regarding

development. And in this collection of questions, where does the school system come into play?

The Ukrainian school system is currently charged with a difficult task. After years of dominance, years of the suzerainty of an outside language, Ukrainian schools must now decide how much emphasis will be placed on Ukrainian as a viable language. The issue is much more complex than it was in the nineteenth century when France was charging its school system with the task of unification through linguistic standardization. We are now living in an age of globalization, where the walls of nationhood are not as impenetrable as they once stood. With the flow of information technology and the breakdown of borders, languages are becoming valuable commodities in much different ways. English is now the global tongue- it is the language of the *world* market, the language that determines the liberal marketplace of globalization, and the chosen way for those who wish to participate in that market to communicate. Russian may not be a threat for the Ukraine, but English is. As Petherbridge-Hernandez and Raby explain:

despite liberalization measures, Ukrainian may still be threatened with extinction, and the culture may continue to be relegated to a subordinate position...in Dnepropetrovsk, many schools require English or German as mandatory subjects, thereby making Ukrainian voluntary. In Kharkov, many of the bilingual information signs were removed and replaced with Russian signs. Decades of Russification could be irreversible and may further erode the Ukrainian language.⁸³

Ukraine has become a divided country, especially in terms of education. Western Ukraine seems straining to become a model of European Union statehood, a modern European structure where the focus is on European cooperation rather than internal nationalist sentiment. Economy drives much of this new assimilation; the money is in cooperation with the EU and the world community at large. The language of this cooperation is English (in the global sense) and even German (the language of the closest

educational linguistic policies. As Fournier points out, “the Russians...protest...against a perceived *linguistic/cultural* exclusion as an extended group. In other words, Russian resistance tends to be directed at a Ukrainian state language law that does not recognize (empire-generated) hybridity within Ukraine” (emphasis taken from original).⁸⁴ Ethnic Russians living in Ukraine still represent a powerful minority, who are having a difficult time adjusting to their characterization as the new ‘lower culture’ in Ukrainian society.

However, there is hope for a resurgence of Ukrainian nationalist sentiment, especially in regard to educational linguistic policies. according to Tomiak:

educational reforms in Ukraine today go in the direction of ensuring education in the mother tongue for all children of Ukrainian nationality at school and all students of Ukrainian nationality in higher education establishments...structural reforms have to wait for more prosperous times, but the Marxist-Leninist bias has gone and has been replaced by an emphasis on Ukrainian national tradition and culture in the teaching of history, geography, literature, and social studies. New textbooks are replacing the old ones, though it will be some time yet before this will apply to all grades at school.⁸⁵

⁸⁴ Anna Fournier, “Mapping Identities: Russian Resistance to Linguistic Ukrainisation in Central and Eastern Ukraine,” *Europe-Asia Studies*, 54 (May, 2002): 415-433. 415. Fournier argues that Russian resistance is not merely a question of ethnicity, but rather the idea of “an *extended* group which they refer to as the ‘Russophones’ or ‘Russian-speakers’. This constructed group includes the Russophone Ukrainians, and is based on the Russian imperial concept of unity between Russians and Ukrainians.” (415). In other words, the issue is purely of language, and not of ethnic identification. Imperial attitudes in Russia identify Russian-speaking

linguistic education policy, and these directions are based upon choices of alliance. Ukraine can decide to follow the Western model and adopt Western practices in terms of language (English and German), or Ukraine can remain attached to Russia and attune its policy structure to the needs of its large Russian minority. Or, the third option- and this is the most attractive option to those concerned with Ukrainian nationalism- Ukraine can spend its educational capital in promoting a resurgence of Ukrainian linguistic development. The future of Ukrainian linguistic integrity will depend upon the path Ukrainian officials decide to follow in structuring the education system.

Conclusion

The purpose of this paper was to address the issue of nationalism as an influence on education policy, especially in regard to linguistic standardization within the education system. Through familiarization with the definition of nationalism and the different varieties of nationalism (political and cultural), it can be asserted that nationalism plays a large role in how education systems are structured, particularly relating to the role of education in standardizing a national language. Throughout modern European history, it has often fallen to the education systems of varied political structures to both strengthen the 'high culture' meanwhile eradicating the language (and therefore the cultural legitimacy) of any 'lower cultures' attempting to resurrect their language communities in the face of outside dominance. It is hoped that through the historical examples provided and the close examination of the two case studies that a greater understanding of the inextricable connection between nationalism and education has been attained. Nationalism is one of the most powerful and influential forces in the modern world. Nationalism has toppled empires and sparked warfare, and it has encouraged both creation and destruction. Nationalism is a force for both revival and dominance, a force that can lift minorities to a more respected position within a community, while pushing them back down at the same time. Despite pronouncements that nationalism is dead, its effects are still ringing throughout the global community. Nationalism is not dead. It is too

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ORDER AMONG THE CHAOS IN DICKENS' *BLEAK HOUSE*

Ray O'Meara

Abstract

This article explores the theme of chaos in one of Charles Dickens' most critically divisive works, *Bleak House*. While many have maintained that the novel is the least cohesive of his many works, I propose that the convoluted plots and loose narrative structure were developed intentionally with the purpose of providing the readers with the same experience as the characters. *Bleak House* is mainly concerned with its critique of the labyrinthine Courts of Chancery, and Dickens proves his narrative mastery by taking both characters and readers on a whirlwind journey through the Victorian legal system, at one time puzzling and entertaining his audience. In order to achieve his purpose, Dickens takes a number of risks that may have led to many of the negative critiques of his book. At times, he abandons his trademark character-naming system, he experiments with the relatively new genre of detective fiction, and he even defies conventional and medical wisdom at the time through the spontaneous combustion of a pivotal character. *Bleak House* is a truly chaotic book, but is intentionally so. This essay seeks to confirm that rather than simply being a novel that lacks cohesiveness, it is a novel that shows Dickens at the height of his creative prowess, and that uniquely and artistically reflects the legal and social chaos that permeated Victorian England.

ills and historical atrocities (the Reign of Terror or workers' conditions during the Industrial Revolution, for example). On the other hand, anyone having read *Oliver Twist*, *Nicholas Nickleby*, *David Copperfield*, and the like may argue that Dickens is a formulaic novelist, writing of a young man trying to discover his place in the world and overcome hardships. Neither of these answers is wrong. However, this is one reason why Charles Dickens' novel *Bleak House* is so problematic to critics and audiences. It is a book exposing the negative aspects of London society. It is also a book about a young orphan. This time, though, and only this time, Dickens uses a female character as narrator. Also for the first and last time, Dickens uses a dual narrator system. One narrator, Esther Summerson, tells her part of the story from the first person point of view. The other unnamed narrator seems semi-omniscient, but more sarcastic and cynical than most similar narrators do. So the story is another social commentary, combined with a coming-of-age tale. *Bleak House* is also a gothic murder mystery. Although Dickens was clearly familiar with the grotesque, it is also a foray into the world of horror. Charles Dickens, perhaps more than any other author, was known for trying to pack every plot and character that he possibly could into his texts, and he often did this very successfully.

Bleak House, however, is different from Dickens' earlier jam-packed successes. Critics and literary analysts have more conflicting thoughts and mixed emotions about this novel than perhaps any of Dickens' other works. While the editor of the 1964 edition described the book as "the finest literary work the nineteenth century produced in England" (Dickens ix), this opinion has been highly contested. When the serial novel was just recently completed, the September 24, 1853, issue of *The Spectator* ran a scathing review by George Brimley, who claimed:

Bleak House is...chargeable with not simply faults, but absolute want of construction. A novelist may invent an extravagant or an uninteresting plot – may fail to balance his masses, to distribute his light and shade – may prevent his story from marching, by episode and discursion: but Mr. Dickens discards plot, while he persists in adopting a form for his thoughts to which plot is

would feel that way about the book. Readers can recognize the factory system and city-versus-country as being at the center of *Hard Times*. People do not have to wonder about who is at the center of *Oliver Twist*. Yet *Bleak House* confuses readers.

Critics such as Brimley and Donovan seem to expect the center to be something more tangible. They want a main character to follow, or a main location, such as Bleak House itself, where all of the action takes place. In what seems to be a culmination of his earlier novels, as well as a lead-in to his later ones, Dickens deviates from his own established form in order to provide looks at all of the social ills that he fought against in previous works: the ridiculous nature of the legal system, the treatment of children and the poor, the harmful effects that industrialization had upon England, and so on. The result of this experiment initially comes across as rather chaotic. The confusion results, though, not from the author's carelessness, but from his design. It is contrived by Dickens to offer a satirical view of Victorian England's chaotic legal, social, and industrial problems, as well as many others. Claiming that there is no center in this novel is truly discounting Dickens' abilities. After all, it cannot be a mistake that nearly every aspect of the novel, from beginning to end, either contributes to or is affected by chaos and confusion. With that said, a close look at *Bleak House* will prove that although it is not directly apparent, there is a semblance of order in the novel, as everything ultimately connects to the center, the chaos.

The novel's center is not immediately apparent for several possible reasons. Some may argue that Dickens was simply new to the mystery genre, and that he may have lost focus because he was out of his element. That theory does not hold water. For all of its unique aspects, *Bleak House* is still clearly Dickensian. Besides, with established mystery writers such as Wilkie Collins proofreading, it is doubtful that Dickens' lack of practice led to Brimley and Donovan's complaints.

Another source of confusion in *Bleak House* is the novel's unique narrative structure. Breaking away from convention, Dickens opts to have the story told by two narrators. The problem with this method, though, is that the readers never quite feel that they are getting the whole story, or an objective

may have with this narrator is the fact that although he/she often speaks in a terse, matter-of-fact manner, the narrator turns out to be less objective than expected. As critic Shirley Galloway notes, “The tone of the impersonal third-person narrator is variously ironic, urbane, familiar, detached, witty, and, at times, expressive of real anger” (1). Knowing that they are not going to receive the full story from the third-person narrator, readers may rely upon the other narrator, Esther Summerson. However, she proves no more helpful.

Esther, unfortunately, appears to be in very much the same position as the readers. Her character is suddenly placed in a world full of people that she does not know and events that she does not understand. As the reader learns, so does Esther, and vice versa. Also, like all first-person narrators, Esther’s assertions must be taken with a grain of salt. True, readers hardly have reason to doubt Dickens’ first-person narrators, but one must keep in mind Esther’s constant state of confusion. So, between the two narrators, readers are able to piece together clues and follow along with the plot. However, neither narrator can be labeled as “the center of the novel.” In fact, due to their confusing and confused natures, these two characters provide major stumbling blocks when attempting to find the center that Donovan was searching for.

Is Bleak House itself the center of the novel? It does not seem very plausible. While the architecture of the house and the action that takes place there are vital to the story, there are far too many important locations in the novel to label just one *the* center of the tale. The same problem occurs when one declares that the Court of Chancery is the center. Again, while the Court is a large part of *Bleak House*, too many events occur outside of Chancery’s sphere of influence for it to be considered the center.

With all else ruled out, the critic is left with characters. Is Esther the center of the novel? Perhaps it is too presumptuous to ask: if David Copperfield is the center of *David Copperfield*, would it not make sense for Esther Summerson to be the center of *Esther Summerson* rather than *Bleak House*? No, Esther is not the center. In fact, there are many chapters in which she is never even mentioned. John Jarndyce could be the center, if he were not such a flat character (even for Dickens). Some argue that the story

chaos – and that is the answer. A number of statements can and have been made about the novel. The center is Chancery. The center is Bleak House itself. The center is the characters. The center is the fog and the gothic atmosphere. None of these are totally wrong. In fact, one concept unites all of these statements: chaos. Despite Brimley and Donovan’s claims to the contrary, the true center of the novel is chaos, as represented by Chancery, by Bleak House, by the characters, and by the atmosphere.

The first chapter of *Bleak House* describes the High Court of Chancery as a place of pure confusion. Everything associated with the Court reflects chaos and disorder, and instills in the reader a sense of befuddlement. In this stage-setting scene, we meet a number of characters who will have very little to do with the actual plot, but who serve as introductions to the chaos that Dickens plan to unleash later. The curious inhabitants of Chancery include “Chizzle, Mizzle, and otherwise” (Dickens 9), who represent the ambiguity of the system itself. We also meet a member of the Court, Mr. Blowers, whose very name and attitude towards the endless Jarndyce and Jarndyce case reflect the fact that items as important as litigants’ lives are often thrown to the wind by pompous Chancellors.

The most helpful character in chapter one, as far as the theme of chaos is concerned, is Mr. Tangle. Tangle’s name is indisputably Dickensian, reflecting, like Blowers, the role that the character will play. Although he is by no means a major character (in fact, he is one of the more minor ones), Tangle represents the confusion of the English legal system. At the same time, he offers a preview of such important characters as Tulkinghorn and Vholes, who will both appear later as meddlesome lawyers who tangle the plot of *Bleak House*, so to speak. Mr. Tangle, in his two-page appearance, serves more than one function, as we will see later.

Two arguably major characters are introduced in chapter one: the Lord Chancellor and Miss Flite. These characters are not only important for their contributions to the plot of the novel, but also for their contributions to the chaos. The Lord Chancellor is never really discussed as a human being with feelings, emotions, and so on, but he is seen as the individual who resides over the foggy disarray that is Chancery. Perhaps claiming that he is

another form: madness. Flite is one of the only characters in *Bleak House* that the reader can clearly place from the moment of her introduction. She is mad. There can be no confusion about that. After all, she “carries some small litter in her reticule which she calls her documents; principally consisting of paper matches and dry lavender” (7). Flite is still quite ambiguous, though. While mad, she seems to have moments of coherence. So, as with many of the sane characters, the reader is ultimately left to wonder when she is telling the truth and when she is lost in her apparent dementia.

Now it is time to start connecting the dots. Dickens establishes in the first chapter of *Bleak House* that Chancery basically means chaos. This point is especially interesting when combined with the thoughts of author and critic G. K. Chesterton:

Miss Flite is a funny character, like Miss La Creevy, but Miss La Creevy means only Miss La Creevy. Miss Flite means Chancery. The rag-and-bone man, Krook, is a powerful grotesque; so is Quilp; but in the story Quilp only means Quilp; Krook means Chancery. Rick Carstone is a kind and tragic figure, like Sidney Carton; but Sidney Carton only means the tragedy of human nature; Rick Carstone means the tragedy of Chancery. Little Jo dies pathetically like Little Paul; but for the death of Little Paul we can only blame Dickens; for the death of Little Jo we blame Chancery. (Chesterton 152-153)

In *Bleak House*, anything related to Chancery, or any other institution, for that matter, is related to chaos. It is not surprising, then, to discover that these characters, and many more, find themselves unable to escape the many types of chaos found in the novel. Many characters, in fact, take the same position as the reader. They are aware that there are a number of important events going on around them, yet they cannot seem to connect them in a logical way. Some are even quite unaware of their roles, and whether they fall on the side of good or evil. It is important to note also that these characters appear throughout both narrations, and in the Court of Chancery, as well as other locales.

number of occasions, but the most straight-forward passage comes from Chapter Twenty-five, when the semi-omniscient narrator tells us, “Mr. Snagsby cannot make out what it is that he has had to do with. Something is wrong, somewhere; but what something, what may come of it, to whom, when, and from which unthought of and unheard of quarter, is the puzzle of his life” (315). Ironically, at this point in the novel, it is also a puzzle of the reader’s life. Dickens creates a character akin to Doyle’s Watson – one that brings the readers into the story, and gives them voice.

Other characters seem to find comfort in the chaos and confusion that surrounds them, while continuing to puzzle readers. Sir Leicester has unique, if not warped, political prowess and reasoning abilities, but chooses to live in a world of confusion. Sir Leicester “calls steadfastly upon the power of Confusion in order to keep on loving his dishonored lady” (Wilt 296). He finds comfort in confusion, developing an “ignorance is bliss” attitude and physically manifesting his confusion in his stammering speech, an after-effect of his stroke. Described paradoxically as a “glorious spider” (347), the reader initially does not know what to make of Sir Leicester Dedlock. Of course, for many authors and poets, spiders can definitely be glorious. When it comes to Dickens, though, most characters described as arachnids, serpents, reptiles, or raptors are predatory monsters (as will be seen with Mr. Vholes). Sir Leicester can be viewed as mildly villainous, but only because of his position as part of the aristocracy that remains blissfully unaware of suffering innocents such as Jo and the victims of Chancery. At the end of the novel, though, Sir Leicester forgives his wife, and the reader finally realizes what a good man he can actually be, and that his categorization as a kind of scoundrel is due only to his social standing. Chesterton states, “Sir Leicester Dedlock and Mr. Harold Skimpole are alike in accepting with a royal unconsciousness the anomaly and evil of their position” (158). We again see the chaos of the novel affecting the characters themselves. Unlike many Dickensian villains who know that they are doing harm (and often enjoy it), like Squeers and Ralph Nickleby, Sir Leicester is blissfully unaware of his contribution to society’s problems. In fact, the aristocrat wants nothing more than to be able to retreat to the confusion that

of confusion for both the readers and most other characters. Before entering into an analysis of Skimpole's contribution to *Bleak House's* chaotic center, another aspect of the author's writing should be briefly discussed: the Dickensian name. In *Bleak House*, Charles Dickens makes it uncharacteristically difficult for the reader to distinguish good characters from evil ones. The logical reason for this involves the fact that Dickens was venturing into the mystery genre, where many of his previous incarnations would fool no one. The way in which he goes about creating mysterious characters is two-fold, involving character names and the experimental dual-narrator method.

What adds perhaps most to the confusion and mystery of *Bleak House* is Dickens' characterization. One aspect of the author's writing is that it is normally rather simple to distinguish the good from the evil. For many of Dickens' characters, the names alone will indicate whether the reader should side with or against them. A reviewer in the 1849 *Macphail's Ecclesiastical Journal* once wrote, "Generally, Mr. Dickens, as if in revenge for his own queer name, does bestow still queerer ones upon his fictitious creations" (Epstein 237). One of the most well known facts about Charles Dickens is that he did not tend to use common names. In fact, the naming of his creations was a careful and meticulous process, often resulting in ludicrous, yet appropriate monikers. As critic Norrie Epstein observes, "One doesn't need to read the novels to know that Flintwich, Gride, Scrooge, and Squeers are all knaves and misers" (237).

Even if a meaning was not directly apparent, readers of Dickens eventually come to crack "a private code in which certain letters or sounds connoted meanings known only to [Dickens]" (237). For example, the letter "Q" was often used to make ugly names for ugly characters. Quinion is the name of one of young David Copperfield's many adult tormentors, while Quilp (*The Old Curiosity Shop*) is one of the more loathed of Dickens' villains. Quinion is never seen, but Quilp is a deranged dwarf. It seems as though any name with a "Q" represents deformity or ugliness. One of Dickens' most monstrous creations is Wackford Squeers, a grotesque, one-eyed lunatic who delights in torturing his so-called students at Dotheboys

character code no longer applies. Examples can be seen in the cases of Phil Squod and Inspector Bucket. Squod, George Rouncewell's assistant at the shooting gallery, has a name that denotes ugliness, thus remaining consistent with Squeers, Quilp, and the rest of Dickens' "Q" characters. However, although Mr. Squod is physically deformed, he proves himself to be one of the more respectable characters in the novel through his devotion to George, and eventually, the ailing Jo. At the same time, the perceptive Inspector Bucket distinguishes himself from buffoons such as Bounderby and Bumble. Bucket is slightly boastful, but pales in comparison to the villainous "B" characters of other novels.

When naming the characters of *Bleak House*, Dickens, for the most part, threw his own rulebook out the window. Look at the names of the suspects in Tulkinghorn's murder: George, Lady Dedlock, and Hortense. "George" is as tame a name as Dickens ever used. George Rouncewell is the perfect murder suspect not just because of his ambiguous name, but because of his ambiguous nature. Like many of Dickens' first person narrators, it does not take long for Esther Summerson to establish a rapport with the reader. One gets the feeling that she, although not an objective observer, is able to be trusted. She likes Ada Clare, so the reader likes Ada. She likes Caddy Jellyby, so the reader does too. However, Esther does not meet George Rouncewell until much later in the story. Until that time, the reader must rely upon accounts from the omniscient narrator, who offers very few clues as to George's motives or intentions. One cannot even rely upon George's acquaintances for assistance. Until winning the favor of Esther, Jarndyce, and the rest, he seems to divide his time between the devilish Smallweeds and the almost angelic Bagnets.

The other innocent suspect in Tulkinghorn's murder, Lady Dedlock, is also quite ambiguous to the reader. The name Dedlock could easily be misconstrued as the name of a Dickensian villain. However, it is ultimately more appropriate in the sense that both Lady Dedlock and the reader are, for lack of a better term, deadlocked. Lady Dedlock finds herself backed into a corner by her manipulative blackmailer, Tulkinghorn. Her indecision only echoes the reader's possible indecision about her character. For one so

conflicted and confused about Lady Dedlock's identity as the readers are.

The discovery of Hortense as the murderess of Tulkinghorn is one of *Bleak House*'s greatest surprises. Putting aside the farfetched and bizarre plot that Hortense constructs to frame Lady Dedlock, the reader can be even more surprised because Hortense does not fit the mold of a Dickensian villain. Although teeming with villains and monsters, *Bleak House* does not contribute to the list of Dickensian uber-villains. For the most part, the antagonists are transitory. Krook dies relatively early in the story, while Vholes appears rather late. Tulkinghorn also meets his end long before the close of the novel, while Smallweed never really lives up to his villainous potential. Hortense has nowhere near the presence of these four in the story, and does not meet the typical villainous description. She seems to be more a source of comic relief than anything else, until we find that she has killed Tulkinghorn. The sight of Mademoiselle Hortense walking "shoeless, through the wet grass" (231) is hardly the menacing image conjured up by a Dickens villain.

If there is one character in the novel that exemplifies the identity confusion experienced by both the reader and other characters, it is the previously mentioned Harold Skimpole. "Skimpole" is yet another name that tells the reader very little about where the character's allegiances lie. The use of the word "skimp" obviously reveals the character to be cheap, or frugal, but in Dickens' world, monetary issues have little to do with character. One of Dickens' most popular characters, Wilkins Micawber, was eternally indebted, yet good-hearted, amiable, and ultimately heroic. The reader's initial response to Skimpole may be reminiscent of the response to Micawber. After all, if one goes by the seemingly trustworthy John Jarndyce's word, Skimpole and Micawber appear to be quite similar. This name, like George and Dedlock, will reveal very little about the character of Skimpole.

The next step would be to weigh the narrators' opinions on Skimpole. Here we encounter another problem. The omniscient narrator deals very little with Skimpole throughout the story. Thus, one must rely upon Esther's accounts. This reveals yet another problem. Esther can tell us *about* Skimpole. For instance, his interests are reflected in his family life:

It is this apparent love for art that makes Skimpole seem so amiable. Many characters who appreciate the humanities get quite far in novels; they are often heroic and admirable. This is not true of Skimpole, though. In fact, this appealing love of art contributes to his confusion of character. Critic Juliet John claims that “Skimpole is so successful at justifying his aestheticism – reminiscent [...] of certain strands in high Romanticism – that many characters in the novel are unable to condemn his amorality, regarding him as a unique enigma” (158). What John says of the other characters of *Bleak House* also holds true for Dickens’ readers. At certain points in the novel (for instance, when we find that it was Skimpole who introduced Richard to the vampiric Vholes), the reader feels an obligation to dislike the childish Skimpole, yet may refrain. Like the actors whom Harold Skimpole supposedly emulates, he has the “ability to charm and captivate an audience [, but] the reaction he stimulates in the audience is best described as an unwilling, rather than a willing, suspension of disbelief” (159).

This acting leads to a great deal of confusion among the readers and the other characters. In the earlier scenes of the novel, Skimpole’s startling views on such topics as plantations are all but brushed aside by Esther, Jarndyce, and the reader, as being the ramblings of a harmless man with the mind of an adolescent. Like Hortense, he at first seems to serve only as a source of comic relief. He seems detached from the outside world (as was his desire). As G. K. Chesterton observes, “The humour of the earlier scenes is delightful – the scenes in which Skimpole looks on at other people paying his debts with the air of a kindly outsider...or the scene in which he tries to explain the advantages of accepting everything to the apoplectic Mr. Boythorn” (157). However, in developing Skimpole’s character further, Dickens slowly involves him more and more in the affairs of the real world. We want to dislike Skimpole when he is unsympathetic towards the ailing Jo, but we take a step back, unsure. We go through the same process when learning of his connection to Vholes. Of course, Inspector Bucket hits the nail on the head when speaking to Esther about the hopeless Romantic. He tells her, “Whenever a person proclaims to you ‘In worldly matters I’m a child.’

years later), when Skimpole is already dead, that the readers and the other characters (besides Bucket) clearly see his true colors. When Esther picks up Skimpole's diary upon his death, she says, "I never read more of it myself than the sentence on which I chanced to light on opening the book. It was this. 'Jarndyce, in common with most other men I have known, is the Incarnation of Selfishness'" (Dickens 729). By the end of *Bleak House*, Skimpole has caused more confusion than perhaps any other character in the novel. Who is the real Harold Skimpole? What can the reader believe of what he has said? Juliet John goes so far as to conclude that "[t]he problem with Skimpole is that all his fine feeling seems to be reserved for art, not human life – though the fact that we usually see him in company with human beings rather than works of art makes even his feeling for art questionable" (161).

Despite the utter confusion that Skimpole causes, many critics would argue that a good deal of *Bleak House's* characters *are*, in fact, easily labeled as either good or evil. Esther Summerson and Ada Clare, for instance, are obviously heroines, since both names have to do with light, or brightness. However, while the natures of some characters may seem clear, they still contribute to the metaphorical fog that permeates the story. Two of the novel's more unmistakably evil characters come to mind: Vholes and Krook.

Mr. Vholes' personality is, like so many of the author's characters, inferred by his name, his appearance, and his occupation. Vholes' name itself conveys the darkness, horror, and chaos that the character ultimately is found to represent. There is first the name's obvious auditory function. "Vholes" is of course a homonym for "voles," small rodents, closely related to rats. While rodents are not known to represent high moral standing to begin with, Dickens' choice of "vole" is especially interesting, since the animals are cannibalistic. These cannibalistic tendencies, though, are most often apparent when another vole is in a weakened state or caught in a trap (Osborne 2). Certainly, of all of Dickens' villains, Vholes is the true cannibal. After all, one of the most memorable statements that the narrator makes concerning the shady lawyer is, "Make man-eating unlawful, and you starve the Vholeses!" (Dickens 483). It certainly seems as though Vholes is making a meal of the weaker Richard throughout the novel. The disgusting slurps and gasps that

master of the name game. As noted earlier, and as evidenced by his working notes on *Bleak House*, names often underwent incredible amounts of alteration before Dickens was satisfied that they were right for the characters. Why should “Vholes” be any different? In fact, the “hole” in “Vholes” seems eerily simplistic and logical. As a lawyer, a representative of Chancery, Vholes is a representative of chaos. The cannibalistic Vholes is sucking the life out of Richard. Richard’s life is being drawn into chaos. Ultimately, Vholes “gave one last gasp as if he had swallowed the last morsel of this client” (760). This client, Richard, is lost in the chaos of Chancery from this point on. He has been sucked into the black hole of England’s legal system.

What gives Vholes’ character traits away perhaps more than anything else, though, is his appearance. Keeping with his gothic and grotesque themes, Dickens creates in Vholes a true monster: a vampire. The idea of Vholes sucking the life out of Richard can almost be seen as literal, since Vholes’ “relation to Richard Carstone is shown again and again to be that of the vampire to its victim” (Pritchard 443). This macabre theory is repeatedly reinforced by Esther’s descriptions of Vholes. He is described as “a sallow man with pinched lips that looked as if they were cold, a red eruption here and there upon his face, tall and thin...and stooping. Dressed in black, black-gloved...there was nothing so remarkable in him as a lifeless manner” (Dickens 469). Vholes’ blackness and coldness both lend themselves to the theory of Vholes as a kind of vampire, as does the allusion to blood on his face. This does not escape Esther, either, who later states, “I felt as if Richard were wasting away beneath the eyes of this advisor, and there were something of the Vampire in him” (720).

Finally, Vholes’ very occupation makes him an important part of the confusion of *Bleak House*. Vholes is not the first of his kind in Dickens’ world. He is most reminiscent of one of Dickens’ first villainous lawyers, the aptly named Mr. Fogg, of the unscrupulous firm Dodson & Fogg, found in *The Pickwick Papers*. In fact, Vholes is nearly a clone of the “elderly, pimply-faced” Fogg, also dressed all in black (Dickens 228). Slightly more developed than his predecessor, Vholes serves the same function as many of Dickens’ lawyers: he contributes to the muck that is England’s legal system.

portrays Vholes as “the personification of darkness, characteristically smear[ing] the window through which she tries to look on the world” (McMaster 169). At this point, although his name and nature are less than confusing to Esther and the reader, Vholes serves the purpose of the novel by becoming a purveyor of confusion, providing the novel with the chaos that it runs on.

The most infamous example of a *Bleak House* character who merely adds to the madness and confusion is Krook. One of Dickens’ most controversial characters, Krook’s only real purpose in the novel is to hide the truth and to confuse other characters and readers. The rag and bottle shop proprietor does so much more than that, though. He is actually the one character in the novel that contributes to every aspect of chaos that has been or will be discussed in this essay. For now, his connection to Chancery and the madness of Miss Flite should be touched upon. Dickens makes it quite clear that “the descriptions of Krook and his rag and bottle shop are meant to function as a grim moral parallel with the Lord Chancellor and Chancery.... Krook’s shop, in its filth and horror, exemplifies in a concrete, physical way, the true moral nature of the court” (Galloway 3). Like the Lord Chancellor, Krook presides over a domain filled with two-faced individuals (Nemo/Hawdon and Weevle/Jobling), demons (his familiar, Lady Jane), and lunatics (Miss Flite). Krook adds to the theme of madness first introduced by his lodger, Miss Flite, but in a more menacing and grotesque way. The rag and bottle man also begins the work that would be continued by Vholes after his death: he makes the characters’ situations as difficult as possible by distorting reality and withholding information. Surrounded by documents that he apparently cannot read, he insists upon hoarding them, implying an almost supernatural knowledge of their importance. These documents, as well as those in the Courts of Chancery are hidden until the end of the novel. Ousby states that “the characters can rarely attempt to view this dense ‘crowd of objects’ clearly or completely” (385). This “crowd of objects,” though, is vital to the story, and thus makes Krook one of the major agents of chaos in the novel.

Crowds of objects are common sights in the world of *Bleak House*,

not all, architectural structures in one way or another. Krook's shop is not the only building in which Esther encounters the incredible number of objects that warrant Dickensian lists.

In general, "Dickensian lists are homages to order and harmony, such as the pleasing inventory of contents of Mrs. Crisparkle's pantry in *Edwin Drood*" (Epstein 252). In *Bleak House*, however, lists are anything but harmonious, and their items usually serve as indications of chaos rather than order. One of the most bizarre and entertaining of these is found in the Jellyby home. Esther describes what happened when she and Caddy attempted to bring order to Mrs. Jellyby's clutter:

[S]uch wonderful things came tumbling out of the closets when they were opened – bits of mouldy pie, sour bottles, Mrs. Jellyby's caps, letters, tea, forks, odd boots and shoes of children, firewood, wafers, saucepan-lids, damp sugar in odds and ends of paper bags, foot-stools, blacklead brushes, bread, Mrs. Jellyby's bonnets, books with butter sticking to the binding, guttered candle-ends put out by being turned upside down in broken candlesticks, nut-shells, heads and tails of shrimps, dinner-mats, gloves, coffee-grounds, umbrellas...(373).

Although Dickens is clearly getting paid by the word, this incredible list lends itself to the continuous theme of chaos and confusion in *Bleak House*. The Jellyby household is actually so chaotic and cluttered that Mr. Jellyby has given up hope of ever changing it, and Caddy simply cannot wait to leave. Other households such as Skimpole's are also in disarray. Esther says, "It was in a state of dilapidation quite equal to our expectation. Two or three of the area railings were gone; the water-butt was broken; the knocker was loose; the bell-handle had been pulled off a long time...and dirty footprints on the steps were the only signs of its being inhabited" (523). This partial description of Skimpole's home again shows the character's overwhelming contribution to the chaos of the story. The passage also continues a theme that sees poor Esther finding herself lost in muddle and confusion throughout the novel.

resulting from a Dickensian *deus ex machina*. The most obvious of Bleak House's confusing traits, though, is its architecture. Related to the chaotic worlds of Chesney Wold, the slum Tom-all-Alone's, Chancery, and the other places that Esther will visit, Bleak House is literally a maze. Esther's first impressions of Bleak House read like passages out of Walpole. Dickens uses the gothic labyrinth theme to give form to Esther's (and the reader's) confusion. Esther tells the reader that "if, instead of going out at Ada's door, you came back into my room, and went out at the door by which you had entered it, and turned up a few crooked steps that branched off in an unexpected manner from the stairs, you lost yourself in passages, with mangles in them" (62). Ironically, though Esther quickly makes herself at home in Bleak House, every resident of the estate will soon find themselves lost in passages, so to speak. Esther will become lost in the mystery of Lady Dedlock. Ada, and especially Richard, will find themselves drawn into the chaos of Chancery. Even John Jarndyce is taken in by the two-faced Skimpole, only to have his generosity repaid with disdain. The idea that *Bleak House's* characters are even controlled by the forces of chaos comes into play in the description of the architecture. Later in the same passage, Esther tells the reader that "if you came out at another door (every room had at least two doors), go straight down to the hall again by half-a-dozen steps and a low archway, wondering how you got back there, or had ever got out of it" (63). Throughout the novel, Esther, Ada, and Richard, as well as those who do not reside in Bleak House, such as Snagsby and Jo, find themselves almost supernaturally guided into awkward and dangerous positions, either by incomprehensible events or sinister forces such as Vholes and Tulkinghorn.

Similar events occur at the other end of the spectrum, Chesney Wold. The Dedlocks' estate is akin to Bleak House in that it is virtually isolated from the rest of the novel's world. Only a certain group of characters are found there, and many, including Volumnia Dedlock, Watt, and Rosa, never seem to have much to do with the outside world. Like that of Bleak House, the architecture of Chesney Wold contributes to the confusion that is central to the novel. One item in particular, the Ghost's Walk, is one of the

Chancery. In the first chapter, readers are introduced to Chizzle, Mizzle, and the rest of the Chancery crowd. As a parallel to Chancery, Chesney Wold hosts a number of guests at all times, including a number of supposed “cousins.” The similarity between the Lord Chancellor’s court and Sir Leicester’s strikes the reader upon introduction to the guests, who besides Volumnia, include Lord Boodle, the Duke of Foodle, and Noodle. In fact, chapter forty echoes the court cases found in Chancery by beginning with the differences that have arisen between party members Lord Coodle and Sir Doodle. Naturally, these interludes, as well as Volumnia Dedlock’s attempts to make sense of the –oodle matters, serve as comic relief, but they also link two distinctive worlds with a common bond: chaos.

Krook’s shop, Jellyby’s house, Bleak House, and Chesney Wold all contribute to the confusion of the novel, but the darkness does not stop there. In *Bleak House*, confusion, darkness, and chaos are also found on a much larger scale. As Allan Pritchard states, “The confusing intricacy of the traditional setting of Gothic fiction is represented in *Bleak House* not so much by any single building as by the vast complex structure of the city as a whole” (439). Anyone at all familiar with Dickens’ London will recognize it as an intimidating maze of streets, many of which belong to unsavory sections such as *Bleak House*’s Tom-all-Alone’s. However, Dickens does not rely solely upon the architecture of the city to make it the home of chaos and darkness. For this, Dickens actually uses another gothic convention: fog. As infamous as London fog is, it has never been used in such a way as it is in *Bleak House*. Dickens relies so much upon the fog throughout the novel that not only does it affect every character in one way or another, but it seems to take on a life of its own and become a character itself.

Fog completely envelopes the novel. From the very beginning readers see nothing but fog, and the view is seldom made clearer. Ironically, Dickens could not be clearer in establishing the importance of fog in the first chapter of the novel:

Fog everywhere. Fog up the river, where it flows among green aits and meadows; fog down the river, where it rolls defiled among the tiers of chimneys and the waterside pollutions of a great

As the passage goes on in the same manner, readers cannot help but envision the fog, establishing it as a vital part of the novel's theme. The descriptions of the gas, mud, mire, and other undesirable elements that engulf the cityscape create an eerie opening quite unlike even the darkest parts of *Hard Times* or *Oliver Twist*. The landscape seems foreign, hellish, and primitive. Dickens even hearkens back to prehistoric times when claiming that "it would not be wonderful to meet a Megalosaurus, forty feet long or so, waddling like an elephantine lizard up Holborn Hill" (5). In order to tell his story properly, Dickens had to do what he could to distort London into an even darker, more confusing realm than usual. The author attempts to create a world "in which normal divisions of time, space, and species seem to have collapsed into each other...a world where categories are constantly crossing, where what seems familiar becomes strangely unfamiliar...where the inhabitants are sometimes so baffled by the world around them that they can no longer survive in it" (Wright 98). He accomplishes this task throughout the novel by the use of fog as a leitmotif.

The "fog and unnatural darkness" (Ousby 382) that is found in the initial description of the city affects and is aided by a number of characters in the novel. In the first chapter, for instance, the previously mentioned Mr. Tangle's speech seems to reflect his dingy surroundings, as he continuously address the Lord Chancellor as "Mlud" (Dickens 9). Perhaps the most bizarre, sickening, yet disturbingly comical scene in *Bleak House*, or any of Dickens' novels, involves the fog, as well as quite a few principle characters. The highly controversial death of Krook by spontaneous combustion shows the grotesque shopkeeper's final contribution to the chaos of the novel. When the reader first meets Krook, his connection to the fog is made obvious while his grizzly death is foreshadowed, as Esther notes "the breath issuing in visible smoke from his mouth" (49). Krook exudes chaos. He not only keeps hidden the documents that will eventually end the case of Jarndyce and Jarndyce, he not only houses an incognito Captain Hawdon, but he seems to physically produce some of the city's fog in a supernatural manner. This characteristic proves no different in death. In a strange perversion of the Biblical "ashes to ashes, dust to dust," the gruesome Krook returns to the

seems to. Shortly after the narrator tells the reader of Snagsby's suspicions concerning the rag and bottle shop, the hapless law stationer takes part in one of the vilest scenes ever written by Dickens. Snagsby, while talking to Weevle, pauses "to sniff and taste the air a little" (394). This smell and taste, attributed by the pair to chops at the Sol's Arms, is apparently really the newly transfigured Krook. Snagsby, who has constantly been lost in the fog, is now horrifically linked and associated with the chaos.

Weevle and Guppy likewise have been involved with the confusion of the novel up until this point, and, like Snagsby, they seem to officially become connected to this fog on the evening of Krook's passing. Weevle was with Snagsby when he observed the thickness and unnaturalness of the air: "Why, there's not much air to be got here; and what there is, is not very freshening...I noticed myself that there is a queer kind of flavour in the place to-night" (393-394). The readers are led to believe that Weevle, too, ingested some Krook, but are thankfully spared the details. Meanwhile, Weevle's partner, Guppy, finds himself in a sullied state, resulting in attempts to, in a sense, wash the death from his hands. Krook, in his new nebulous form, is able to find his way into Weevle's lodging and onto Guppy himself. From this point on, Guppy is completely immersed in the chaos of the novel, encountering characters from every end of the spectrum, from the Smallweeds to Lady Dedlock, and becoming involved in every one of *Bleak House's* mysteries. Krook's combustion serves not only as a physical manifestation of the fog's consumptive power, but as a rite of initiation for the likes of Snagsby, Weevle, and Guppy, who are now completely immersed in the chaos.

It seems fitting to end this essay with the chaos that ends the novel. To complete the theme started by the actual London fog and the confusion of Chancery in the first chapter, and continued by characters such as Krook and Jellyby, Dickens ends the novel with still more mayhem. The final chapter of *Bleak House* is quite unlike that of most other Dickens novels. Critic John Kucich states the popular belief that "Dickens's endings are so commonly seen as expressions of a wish for stasis" (96). Dickens does often attempt to wrap his stories up neatly, with a minimum of confusion, and almost no room

Once again, *Bleak House* proves an exception to the rule. In this book, the “ending never really ends” (101). One unusual feature about this ending, especially for Dickens, is that it is split into two parts, due to the presence of the dual narrators. Each presents a chapter of conclusion, but even between the two of them, the reader can still feel unsatisfied. In the first of the concluding chapters, the reader is given a final tour of Chesney Wold, and bids farewell to characters such as Boythorn and Sir Leicester. In the next chapter, Esther provides information on what happened to Charley, the Jellybys, the Turveydrops, and of course Ada, Jarndyce, and Woodcourt. What about such memorable characters as the Chadbands, the Bagnets, and above all, Mr. Bucket? Grandfather Smallweed, who had the potential to become the next great Dickens villain is simply sent away dissatisfied – a far cry from the fiery fate of Miss Havisham or those of Sikes and Fagin. In a manner uncharacteristic of Dickens, these characters say their farewells unceremoniously, never to be mentioned again. At the same time, it seems that every character associated with Chancery disappears into the fog, along with Jarndyce and Jarndyce. Kenge fades into the distance along with Vholes, who is last seen appropriately in the shadows, after the famed case is brought to a close. Guppy and Jobling/Weevle are last seen after Esther’s rejection of the former, but anyone wondering what became of them is left guessing. The Lord Chancellor is never mentioned again. He, like his counterpart Krook, vanishes into the fog. Thus, the fog found in Chancery, and throughout the novel, continues to confound the reader to the very end.

Is there hope for freedom from this fog, or for the characters to sort out the confusion of their situations? Dickens does seem to leave some small glimmer of hope. Although the majority of *Bleak House*’s characters are lost and confused, a select few manage to find their way out of the darkness and bring order to the chaos, much as Dickens does in the construction of his plot. Esther, and arguably Ada, is able to bring stability to the labyrinth that is Bleak House, while Bucket succeeds in solving one mystery after another. One must keep in mind the many casualties of chaos, though. Richard is sucked into the void of Chancery, Miss Flite loses her mind at the hands of the legal system, and Jo and Lady Dedlock die because of societal ills.

Ultimately, the confusion that arises from *Bleak House* may be attributed to the author attempting to create a culmination of his greatest works. While other great novels, such as *Great Expectations*, would follow, Dickens seems to bring all of his favorite themes together in *Bleak House*, and the idea of confusion seems to fit them all. While confusion was never a main theme in his orphan stories (*Oliver Twist*, *Nicholas Nickleby*, *David Copperfield*), it was always present. Who could the narrators trust? What did the future hold? These stories were riddled with uncertainty, but in a subtler way than the story of Esther Summerson. Other works focused on the horrors of institutions. *The Pickwick Papers* marked Dickens' first legal satire, introducing Vholes' forerunners, Dodson and Fogg. The chaos of the legal system, and other institutions (such as the educational system) is also reflected in the plots of his earlier works. In *Bleak House*, Dickens seems to have found a way to link his popular themes. Although many critics such as Brimley and Donovan may fail to see a method to *Bleak House*'s seeming madness, Dickens truly writes this novel out of what A.O.J. Cockshut refers to as his "new respect for order" (963). If Dickens gained anything from his own writing, it was a heightened feeling of disdain for the chaos of the legal system, the educational system, and the Victorian treatment of orphans, whose lives could be as chaotic as any of his plots. By devoting narration, character, architecture, and the gothic techniques of the mystery writer to the exploration of the various types of chaos in England, Dickens was able to make the ultimate stand against the various institutions that he had opposed throughout his career. While doing this, he completed the transition into his mature novels and successfully experimented with the gothic mystery genre. *Hard Times*, *Great Expectations* and *The Mystery of Edwin Drood* would revisit his classic themes, but *Bleak House* is certainly the book most centered on, and most critical of, chaos.

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ANALYSIS OF ITERATIVE METHODS FOR SOLVING MATRIX EQUATIONS

Samantha Bourque-Trieff

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that can be modeled using the Helmholtz equation. This partial differential equation is often difficult to solve and can be approximated using the finite element method. The finite element method is a matrix equation of the form $Ax=B$. When direct methods of solving this equation become too lengthy and inefficient, iterative methods can be utilized to find a solution to the given equation. A variety of methods have been studied carefully so that suggestions of the best method to use to solve the matrix equation formed by the finite element method can be made. These methods have been programmed and compared so that the suggestions can be supported by both graphical and algebraic evidence.

Problems often arise in the physical world that can be thoroughly examined and better understood through the use of mathematical models. However, as technology progresses beyond unimaginable bounds, there are several types of problems that will become more sophisticated and difficult to solve. In the past, many of these problems could be solved using direct and straight-forward methods. However, as they become more complex, these problems will require more intricate techniques for finding solutions and computers that have the capability of processing a lot of information at very rapid speeds.

Scattering theory, fluid mechanics, and physical engineering are just some of the areas that provide opportunities for mathematical applications to physical problems. Scattering theory is a very common and practical area of concern. The ways in which waves scatter after coming into contact with various objects are useful in many aspects of life. For example, medical imaging uses different types of waves, such as X-rays, ultrasound waves, and electromagnetic waves, in many facets of study. Seismic waves are often used to evaluate the structure of the Earth while X-rays are used to study the structure of the human body. Most of these waves can be considered time-independent and can be modeled using the Helmholtz equation (Froese 1). In fact, the finite element method is often used to model the Helmholtz equation in these applications.

Fluid mechanics and physical engineering are other fascinating areas of concern. The flow of fluids is a very interesting and useful topic of study, which spans areas such as blood and water. It is also common to study the structural integrity of buildings in the area of physical engineering. The finite element method is widely used to study both fluid mechanics and areas of physical engineering. Clearly, with so many important applications, the Helmholtz equation and the finite element method should be studied carefully and used to solve many of the problems arising in the physical world.

The Helmholtz equation is a partial differential equation that models the motion of many types of waves, most commonly acoustic waves. It is an

Restricting the space \mathbf{R}^n makes the problem more specific and easier to handle even though some of the physical meaning of the problem is lost. Nevertheless, the Helmholtz equation will be limited to a one-dimensional consideration of \mathbf{R}^n , namely \mathbf{R} . The Helmholtz equation rarely has a solution that can be expressed easily or explicitly. As a result, it is necessary to seek an approximation to the true solution.

The finite element method is one such method of approximating the true solution to the Helmholtz equation. Instead of considering the equation over the defined physical domain, the finite element method partitions the domain and then approximates the solution locally over each subdomain.

Historically the finite element method has been a widely used method of approximating solutions to partial differential equations. The origin of the method is difficult to find because it is the result of multiple lines of work in various areas of mathematics. Commonly, the finite element method is traced to work by Courant in the 1940's on piecewise linear approximations using triangles. However, the use of approximations of variational problems can be found in the work of Schellback in the late 19th century (Oden 125). Nevertheless, the finite element method took giant leaps during the 1960's and 70's. The success of the method in problems with both symmetric and asymmetric operators in these later years led to a rise in its popularity (Oden 126). The use of the finite element method has continued to grow and is now used widely in mathematics.

There are advantages of using the finite element method in various areas of mathematics. Finite elements can be applied to domains with arbitrary shapes and boundary conditions that allow for use on a large variety of domain areas. Similarly, the finite elements can be applied anywhere in the domain to facilitate finding solutions, and the use of the variational, or weak, formula provides the proper setting for the existence of irregular solutions that may or may not be found using other approximation methods (Oden 560). These traits make the finite element method a good choice for

where A is a known $n \times n$ square matrix, b is a known n -dimensional vector, and x is an unknown vector variable. Once this equation has been formed, the problem then becomes how to find its solution. Simple problems allow for direct methods, such as multiplying both sides by A^{-1} to arrive at the solution of

$$x = A^{-1}b. \quad (1.3)$$

However, as problems become more complex, the number of unknown variables becomes great, and direct methods of solving the matrix equation can become unwieldy or impractical. Computers, as powerful as they may appear, are incapable of storing enough memory to solve equations that require such enormous attention. While the practical applications of these problems are essential to everyday life, the solutions are hard to come by. Therefore, other methods of finding a solution must be considered.

Another class of techniques, called iterative methods, usually provides viable alternatives to approximate the solution to a degree of negligible error. Many iterative methods lead to reasonable solutions to the matrix equation; however, the manner in which the solution is found is of the utmost concern. It is necessary to consider techniques that are efficient and accurate, keeping in mind that the implementation of any algorithm is limited by the amount of memory and time available.

In general, iterative methods begin with an initial approximation and all subsequent approximations are then constructed based on the accuracy of the previous approximations (Burden 418). The error can be measured after each successive estimate and used to determine the level of precision of the approximation. If this error is greater than desired, approximations continue to be created until the desired level of accuracy is achieved.

While all iterative methods are based on the same premise, they

number of iterations required for convergence. Simple problems will be used to exemplify these ideas, and the differences will become very clear when the convergence rates are compared both algebraically and graphically. It must be noted that the difference between these convergence rates will seem trivial at first. However, physical applications of any kind, including those of acoustic waves, will be much larger and more complex than the examples provided. The almost seemingly insignificant difference between convergence behaviors of the iterative methods will be exaggerated greatly when applied to these real-world problems.

While many iterative techniques may provide accurate answers, the amount of time required to use some techniques outweighs the benefits of solving the problem. In fact, computers are sometimes incapable of handling a problem of such immense proportion; solutions to important problems may never be found. Therefore, it is necessary to find ways to cut down on computer memory and processing time in order to ensure the possibility of finding solutions.

In order to facilitate the process of finding a solution to equation (1.2), characteristics of the matrix A , such as being positive definite and strictly diagonally dominant, can easily be exploited. For example, simple iterative methods are often the first choice for solving the problem because they are straightforward and fairly easy to use. However, the simple methods require that the matrix A satisfy various conditions, which are often not fulfilled in real-world applications. Thus, using the characteristics of A to eliminate futile methods will ultimately save time and energy.

It is essential to test the various techniques for specific problems to verify which method works best for different types of problems. *Maple* computer programs have been constructed to test the various techniques. The results of these programs have been used to compare the methods and the efficiency of each. A program was created to first compute the results of the finite element method in the form of equation (1.2). Similarly, a series of programs have been produced to find approximate solutions using the various iterative techniques. Finally, these solutions are compared in terms of

will explore many of these techniques and suggest ways in which to select the best method for the specific problem at hand. In what follows, a thorough explanation of the finite element method will be followed by outlines of various iterative techniques and the results they produce. Finally, a detailed discussion of the advantages and disadvantages of each method will culminate in suggestions for ways to solve the Helmholtz equation in the most accurate and efficient manner possible.

2. Finite Element Method

The finite element method provides the means for finding approximate solutions to many partial differential equations. The unwieldy space is partitioned and considered over finite elements, making the problem more manageable. A matrix-vector equation is produced as a result of the finite element method, which can then be solved using a variety of techniques. The algorithmic process of the finite element method can easily be programmed and used to solve a variety of differential equations that result from problems in the physical world.

The problem considered here is to determine the function $u(x)$ for $x \in \mathbf{R}$ satisfying the differential equation

$$u'' + cu = 0 \tag{2.1}$$

on the interval $D=[a,b]$, where it is required that the function satisfy the boundary conditions

$$u(a) = u_a \tag{2.2}$$

and

step is to multiply both sides by v , where v is an element of some function space, V . This results in

$$u''v + cuv = 0. \tag{2.4}$$

The next step is to integrate this new equation over the interval D . That is,

$$\int_a^b u''v + \int_a^b cuv = 0. \tag{2.5}$$

Using integration by parts on the first term of this equation and simplifying the result leads to

$$vu' \Big|_a^b - \int_a^b u'v' + c \int_a^b uv = 0. \tag{2.6}$$

This equation can be rearranged and simplified to arrive at the equation

$$c \int_a^b uv - \int_a^b u'v' = -vu' \Big|_a^b, \tag{2.7}$$

which can be written as

$$c(u, v) - (u', v') = \lambda(v), \tag{2.8}$$

and (\cdot, \cdot) is the usual L_2 -inner product. Typically the left side is written as

$$A(u, v) = c(u, v) - (u', v'), \quad (2.10)$$

which leads to the problem of finding $u \in V$ so that

$$A(u, v) = \mathbf{L}(v) \text{ for all } v \in V. \quad (2.11)$$

The finite element method then seeks an approximation to the variational problem from a finite dimensional subspace of V .

The approximation begins by partitioning D , the physical domain of the problem. A partition of D is a collection of subsets, $\{K_i\}$, where each K_i is open, $K_i \cap K_j$ is the empty set, for $i \neq j$, and $\overline{D} = \bigcup_i \overline{K}$. Since the problem has been limited to a one-dimensional consideration of the Helmholtz equation, the K 's can be defined as

$$K_k = (x_{k-1}, x_k), \quad k = 1, \dots, N, \quad (2.12)$$

where N is the number of subintervals, and the length of each subinterval can be defined as

$$h_k = x_k - x_{k-1}. \quad (2.13)$$

The global basis functions are taken to be linearly-independent, continuous functions that span the subspace, V . Typically these functions are linear and satisfy

$$\mathbf{f}_i(x_j) = \mathbf{d}_{ij}. \quad (2.14)$$

One choice might be

$$\mathbf{f}_k(x) = \begin{cases} \frac{x - x_{x-1}}{x_k - x_{k-1}}, & x \in K_k \\ \frac{x - x_{k+1}}{x_k - x_{k+1}}, & x \in K_{k+1} \\ 0, & x \notin K_k \text{ and } x \notin K_{k+1} \end{cases}. \quad (2.15)$$

The polynomials that are non-zero on each K_k can then be defined locally in the following manner:

$$\mathbf{f}_{k-1} \Big|_{K_k} = \frac{x - x_k}{x_{k-1} - x_k} \quad (2.16)$$

and

$$\mathbf{f}_k \Big|_{K_k} = \frac{x - x_{k-1}}{x_k - x_{k-1}}.$$

the reference element, denoted $(\hat{K}, \hat{P}, \hat{\Sigma})$. A mapping, T , exists such that $T_k : \hat{K} \rightarrow K_k$, where this mapping relates the reference element \hat{K} to each K_k . In fact, it is common, when working in \mathbf{R} , to choose the reference element \hat{K} to be $\hat{K} = [-1,1]$.

The mapping is an affine transformation given by

$$T_k(\hat{x}) = a\hat{x} + b, \quad (2.18)$$

such that

$$T(-1) = x_{k-1} \quad (2.19)$$

and

$$T(1) = x_k. \quad (2.20)$$

It turns out that this function is

$$T_k(\hat{x}) = \frac{x_k - x_{k-1}}{2} \hat{x} + \frac{x_k + x_{k-1}}{2}. \quad (2.21)$$

Defining x to be $T_k(\hat{x})$, equation (2.21) can be solved for \hat{x} to get

(2.22)

This equation can be defined as $\hat{T}_k(x)$ so that

$$\hat{x} = \hat{T}(x), \quad (2.23)$$

where

$$\hat{T}(x) : K_k \rightarrow \hat{K}. \quad (2.24)$$

Thus, the appropriate mappings have been defined as to create a relation between K and \hat{K} .

This relation between K and \hat{K} leads to definitions for \mathbf{P} and Σ . In particular, the basis for each $P_{ik} = \mathbf{P}_k(K_i)$ can be found by mapping the basis for $\hat{P}_i = \mathbf{P}_k(\hat{K}_i)$. For example, if $\{\hat{f}_0(\hat{x}), \hat{f}_1(\hat{x})\}$ represents a linear basis on \hat{K} , where,

$$\hat{f}_0(\hat{x}) = \frac{1}{2}(1 - \hat{x}) \quad (2.25)$$

and

$$\hat{f}_1(\hat{x}) = \frac{1}{2}(1 + \hat{x}),$$

$$\mathbf{f}_{k-1}|_{K_k} = \mathbf{f}_1(T_k(x)) \quad (2.27)$$

and

$$\mathbf{f}_k|_{K_k} = \hat{\mathbf{f}}_0(\hat{T}_k(x)), \quad (2.28)$$

which are equivalent to equations (2.16) and (2.17) respectively. If a higher order approximation is desired, a few changes must be made because all of the basis functions cannot be associated with a specific vertex.

The basis functions have already been looked at from three different perspectives: global basis functions, global basis functions restricted to K_k , and basis functions for \hat{P}_k . A difficulty with linear functions lies in the fact that they, when defined globally and left unrestricted, do not have support contained in a single K_k . Support may exist when the functions are considered over the two intervals which share the vertex with which they are associated. However, it is less difficult to define linearly-independent global basis functions of order greater than one so that the support exists more naturally within a single K_k . The problem with higher-order functions is that they are usually not associated with a vertex, requiring a more detailed notation. Using the mapping that is described in equation (2.24), consider defining the function

$$\hat{\mathbf{f}}_2(\hat{x}) = \frac{1}{2}(\hat{x}^2 - 1) \quad (2.29)$$

where the fact that $\hat{\mathbf{f}}_2(-1) = \hat{\mathbf{f}}_2(1) = 0$ guarantees that

$\hat{\mathbf{f}}_{k2}(x_{k-1}) = \hat{\mathbf{f}}_{k2}(x_k) = 0$. In general, if $\hat{\mathbf{f}}_p(\hat{x})$, for $p > 1$, satisfies

$$\hat{\mathbf{f}}_p(-1) = \hat{\mathbf{f}}_p(1) = 0, \quad (2.31)$$

then the global basis functions can be defined so that

$$\mathbf{f}_{kp}(x) \Big|_{K_k} = \hat{\mathbf{f}}_p(\hat{T}_k(x)). \quad (2.32)$$

In fact, it is often the case that the global basis functions are described in terms of a reference element basis.

There are many choices for a basis of $\mathbf{P}_p(\hat{K})$. However, it is common to choose the following basis:

$$\hat{\mathbf{f}}_0(\hat{x}) = \frac{1}{2}(1 - \hat{x}) \quad (2.33)$$

$$\hat{\mathbf{f}}_1(\hat{x}) = \frac{1}{2}(1 + \hat{x}) \quad (2.34)$$

$$\hat{\mathbf{f}}_p(\hat{x}) = \int_{-1}^{\hat{x}} L_{p-1}(s) ds, \quad p \geq 2,$$

Once the basis for \mathbf{P} has been found, the problem can be written in terms of this basis. That is, given a global basis $\{\mathbf{f}_i\}_{i=1}^n$, u can be written as

$$u = \sum_{i=1}^n \mathbf{a}_i \mathbf{f}_i, \quad (2.36)$$

which leads to

$$c \left(\sum_{i=1}^n \mathbf{a}_i \mathbf{f}_i, v \right) - \left(\sum_{i=1}^n \mathbf{a}_i \mathbf{f}_i, v' \right) = \mathbf{L}(v). \quad (2.37)$$

For each choice of v , there is a need for only one equation. However, v can be written as

$$v = \mathbf{f}_j, \quad j = 1, \dots, n, \quad (2.38)$$

resulting in n different equations. Therefore, the problem can be written as

$$c \left(\sum_{i=1}^n \mathbf{a}_i \mathbf{f}_i, \mathbf{f}_j \right) - \left(\sum_{i=1}^n \mathbf{a}_i \mathbf{f}_i, \mathbf{f}_j' \right) = \mathbf{L}(\mathbf{f}_j) \quad (2.39)$$

or

$$(2.41)$$

and

$$S = (\mathbf{f}'_i, \mathbf{f}'_j), \quad (2.42)$$

the problem can be written as a matrix equation

$$Q\bar{\mathbf{a}} = (cM - S)\bar{\mathbf{a}} = \mathbf{L}, \quad (2.43)$$

where

$$\mathbf{L} = \mathbf{L}(\mathbf{f}_j). \quad (2.44)$$

In order to understand the left side of the equation better, it is useful to look at M and S individually. In the case of uniform linear approximations, the entries for the matrices M and S are defined as

$$M_{ij} = \int_D \mathbf{f}_i(x)\mathbf{f}_j(x)dx \quad (2.45)$$

and

$$S_{ij} = \int_D \mathbf{f}'_i(x)\mathbf{f}'_j(x)dx. \quad (2.46)$$

$$M_{ij} = \begin{cases} \int_{K_{\max(i,j)}} \mathbf{f}_i(x) \mathbf{f}_j(x) dx, & i \neq j \\ \int_{K_{i-1}} (\mathbf{f}_i)^2(x) dx + \int_{K_i} (\mathbf{f}_i)^2(x) dx, & i = j \end{cases} .$$

(2.47)

Similarly, S can be defined as

$$S_{ij} = \begin{cases} \int_{K_{i-1}} (\mathbf{f}_i)^2(x) dx + \int_{K_i} (\mathbf{f}_i)^2(x) dx, & i = j \\ \int_{K_{\max(i,j)}} \mathbf{f}_i(x) \mathbf{f}_j(x) dx, & i \neq j \end{cases} .$$

(2.48)

Using the change of variable formula, the entries for M and S can be defined in terms of the basis on the reference element. That is, using the formula

$$\int_a^b f(x) dx = \frac{b-a}{2} \int_{-1}^1 f\left(\frac{(u-1)(b-a)}{2} + b\right) du ,$$

(2.49)

where

$$u = \frac{2}{b-a}(x-b) + 1 ,$$

(2.50)

it can be shown that

(2.51)

and

$$\int_{K_k} \mathbf{f}_k(x) dx = \frac{h_k}{2} \int_{\hat{i}} \hat{\mathbf{f}}_0(\hat{x}) d\hat{x}.$$

(2.52)

Therefore, if the reference element is defined as $[-1,1]$, then the entries for M can be defined as

$$M_{ij} = \begin{cases} \frac{h_{i-1}}{2} \int_{-1}^1 \frac{1}{2} (1 + \hat{x}) d\hat{x} + \frac{h_i}{2} \int_{-1}^1 \frac{1}{2} (1 - \hat{x}) d\hat{x} = \frac{h_{i-1} + h_i}{3}, & i = j \\ \frac{h_{\min(i,j)}}{2} \int_{-1}^1 \frac{1}{2} (1 - \hat{x}) \frac{1}{2} (1 + \hat{x}) d\hat{x} = \frac{h_{\min(i,j)}}{6}, & i \neq j \end{cases}$$

(2.53)

and the entries for S can be defined as

$$S_{ij} = \begin{cases} \frac{h_{i-1}}{2} \int_{-1}^1 \left(\frac{1}{2} \hat{x}\right) d\hat{x} + \frac{h_i}{2} \int_{-1}^1 \left(-\frac{1}{2} \hat{x}\right) d\hat{x} = \frac{h_{i-1} + h_i}{4}, & i = j \\ \frac{h_{\min(i,j)}}{2} \int_{-1}^1 \left(-\frac{1}{2} \hat{x}\right) \left(\frac{1}{2} \hat{x}\right) d\hat{x} = -\frac{h_{\min(i,j)}}{4}, & i \neq j \end{cases}.$$

(2.54)

When higher order approximations are used, the entries for M and S

basis functions. The entries of Λ_i will be zeroes and ones, designating the relationship between each local basis function and all of the global basis functions that are non-zero on K_k . This matrix can then be used to construct the entries of the matrices M and S in a very easy manner. That is, the entries of M can be found by

$$M = \sum_{i=1}^N \Lambda_i m_i \Lambda_i^T, \quad (2.55)$$

where

$$m_i = \frac{h_i}{2} \hat{m} \quad (2.56)$$

and

$$\hat{m} = \int \hat{\mathbf{f}}_i(\hat{x}) \hat{\mathbf{f}}_j(\hat{x}) d\hat{x}. \quad (2.57)$$

Similarly, the entries of S can be defined as

$$S = \sum_{i=1}^N \Lambda_i s_i \Lambda_i^T, \quad (2.58)$$

where

and

$$\hat{s} = \int \hat{\mathbf{f}}_i'(\hat{x}) \hat{\mathbf{f}}_j'(\hat{x}) d\hat{x}.$$

(2.60)

Using M and S , it is now possible to construct the left side of equation (2.43), using

$$Q = cM - S.$$

(2.61)

Therefore, it only remains to be shown what the right side of equation (2.43) will be.

The right side of equation (2.43) is a vector defined by

$$\mathbf{L} = \mathbf{L}(\mathbf{f}_j).$$

(2.62)

It turns out that this vector will have nonzero entries only in the first and last positions. This is the result of the manner in which the entries of \mathbf{L} are constructed. Recall that $\mathbf{L}(v) = -vu \Big|_a^b$, which can also be written as

$$\mathbf{L}(v) = -vu \Big|_a^b = -v(a)u'(a) + v(b)u'(b).$$

(2.63)

Since only the points a and b are used in evaluating the vector, consider the values of the basis functions at only these two points. It turns out that

and

$$\mathbf{L}(\mathbf{f}_N) = 0 \cdot u'(a) + 1 \cdot u'(b) = u'(b). \quad (2.65)$$

When considering the value of $\mathbf{L}(\mathbf{f}_j)$ at all other basis functions between \mathbf{f}_1 and \mathbf{f}_N , the values for $v(a)$ and $v(b)$ will be zero. Therefore, the right side of equation (2.43) will be

$$\mathbf{L}^T = [-u'(a) \quad 0 \quad \dots \quad 0 \quad u'(b)]. \quad (2.66)$$

This construction of the right side vector utilizes the naturally occurring boundary conditions in the variational form.

Nevertheless, it is also possible to construct the right side vector as

$$\mathbf{L}^T = [u(a) \quad 0 \quad \dots \quad 0 \quad u'(b)]. \quad (2.67)$$

The boundary conditions used in this construction are artificial but can make the problem easier to handle. With the changes made to the right side vector, some changes must also be made to the left side to keep the problem the same. In the first construction of the right side, the problem looks like

$$\begin{bmatrix} \mathbf{M} & \mathbf{M} & \mathbf{O} & \mathbf{M} & \mathbf{M} \\ q_{(n-1),1} & q_{(n-1),2} & \Lambda & q_{(n-1),(n-1)} & q_{(n-1),n} \\ q_{n,1} & q_{n,2} & \Lambda & q_{n,(n-1)} & q_{n,n} \end{bmatrix} \begin{bmatrix} \mathbf{M} \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} \mathbf{M} \\ 0 \\ u'(b) \end{bmatrix}. \quad (2.68)$$

However, when the changes are made to the right side in the second construction, changes must be made to the left side of the equation so that it looks like

$$\begin{bmatrix} 1 & 0 & \Lambda & 0 & 0 \\ 0 & q_{2,2} & \Lambda & q & q_{2,n} \\ \mathbf{M} & \mathbf{M} & \mathbf{O} & \mathbf{M} & \mathbf{M} \\ 0 & q_{(n-1),2} & \Lambda & q_{(n-1),(n-1)} & q_{(n-1),n} \\ 0 & q_{n,2} & \Lambda & q_{n,(n-1)} & q_{n,n} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \mathbf{M} \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} u(a) \\ 0 \\ \mathbf{M} \\ 0 \\ u'(b) \end{bmatrix}. \quad (2.69)$$

With these changes, the initial problem stays the same. Using these definitions for Q and L , equation (2.43) is ready to be solved. It is possible to solve equation (2.1) using this simple matrix equation, which can easily be programmed and solved using a computer.

2.1. Examples

In order to more clearly understand the underlying theory of the finite element method, consider the problem

$$u'' + 4u = 0, \quad (2.70)$$

and

$$u'(4p) = 0. \tag{2.72}$$

The finite element method can be used to efficiently approximate the solution to this problem using a matrix equation.

It turns out that the true solution of this problem can be found using analytical methods of differential equations. The problem at hand is a homogeneous, linear, second-order differential equation of the form

$$m \frac{d^2u}{dt^2} + b \frac{du}{dt} + ku = 0, \tag{2.73}$$

where $m = 1$, $b = 0$, and $k = 4$. Since $b = 0$ the oscillator is said to be undamped, which results in a characteristic polynomial of the form

$$ms^2 + k = 0 \tag{2.74}$$

or, more specifically,

$$s^2 + 4 = 0. \tag{2.75}$$

Since $k > 0$ and $m > 0$, it is common to define

$$\mathbf{w} = \sqrt{\frac{k}{m}}.$$

$$u(t) = k_1 \cos(\omega t) + k_2 \sin(\omega t) \quad (2.77)$$

(Blanchard 302). After applying the boundary conditions in equations (2.71) and (2.72), the solution to equation (2.70) is found to be

$$u(t) = \sin(2t) . \quad (2.78)$$

With this solution in mind, the finite element method can be used and then the results compared to the true solution to determine the accuracy of its method.

Begin by defining the physical element (D, \mathbf{P}, Σ) in the following manner:

$$\begin{aligned} D &: [0, 4\mathbf{p}] \\ \mathbf{P} &: \mathbf{P}_1(D) , \\ \Sigma &: N_i(\mathbf{f}_j) = \mathbf{f}_j(x_i) = \mathbf{d}_{ij} \end{aligned} \quad (2.79)$$

where \mathbf{P}_1 is the set of polynomials with degree of at most 1. The first step in the finite element method is to partition D . Choosing $N = 16$, the physical domain can be partitioned into 16 subintervals of equal length of the form

$$K_k = \left(\frac{4\mathbf{p} - 0}{16}(k-1), \frac{4\mathbf{p} - 0}{16}(k) \right) = \left(\frac{\mathbf{p}}{4}(k-1), \frac{\mathbf{p}}{4}(k) \right) , \quad k = 1, \dots, N \quad (2.80)$$

where the length of each subinterval can be defined as

If the reference element is chosen to be $[-1,1]$, each of these subintervals can be mapped to the reference element using the mapping found in equation (2.24). Since only linear functions are being used, a linear basis of \hat{P} can be defined as in equations (2.33) and (2.34).

Connectivity matrices are needed to construct the matrices M and S and create a mapping between the local and global basis functions. Because only linear functions are being used as a basis, the connectivity matrices will be 17×2 matrices with entries of zeroes and ones. For example,

$$\Lambda_1^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.82)$$

and

$$\Lambda_4^T = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (2.83)$$

These connectivity matrices can be used in conjunction with the matrices m , \hat{m} , s , and \hat{s} to construct M and S . Using equation (2.57), \hat{m} is found to be

$$\hat{m} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & \frac{2}{2} \end{bmatrix}$$

$$m_i = \begin{bmatrix} \frac{12}{24} \mathbf{p} & \frac{24}{12} \mathbf{p} \\ \frac{1}{24} \mathbf{p} & \frac{1}{12} \mathbf{p} \end{bmatrix}.$$

(2.85)

Using these matrices, it is easy to find the matrix M to be

$$M = \begin{bmatrix} p/12 & p/24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p/24 & p/6 & p/24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & p/24 & p/6 & p/24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p/24 & p/6 & p/24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p/24 & p/6 & p/24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p/24 & p/6 & p/24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & p/24 & p/6 & p/24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & p/24 & p/6 & p/24 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p/24 & p/6 & p/24 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p/24 & p/6 & p/24 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p/24 & p/6 & p/24 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p/24 & p/6 & p/24 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p/24 & p/6 & p/24 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p/24 & p/6 & p/24 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p/24 & p/12 \end{bmatrix}.$$

(2.86)

Similarly, S can be found using the matrices s and \hat{s} , which are defined as

$$\hat{s} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

(2.87)

and

Thus, S is constructed to be

$$S = \begin{bmatrix}
 4/p & -4/p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 -4/p & 8/p & -4/p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & -4/p & 8/p & -4/p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & -4/p & 8/p & -4/p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & -4/p & 8/p & -4/p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & -4/p & 8/p & -4/p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & -4/p & 8/p & -4/p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & -4/p & 8/p & -4/p & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4/p & 8/p & -4/p & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4/p & 8/p & -4/p & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4/p & 8/p & -4/p & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4/p & 8/p & -4/p & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4/p & 8/p & -4/p & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4/p & 8/p & -4/p & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4/p & 8/p & -4/p \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4/p & 4/p
 \end{bmatrix} .$$

(2.89)

Now that both M and S have been formed, the matrix Q can be found using the formula

$$Q = 4M - S.$$

Therefore, the matrix Q looks like

$$Q = \begin{bmatrix} 0 & -4.96 & -9.66 & -4.96 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4.96 & -9.66 & -4.96 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4.96 & -9.66 & -4.96 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4.96 & -9.66 & -4.96 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4.96 & -9.66 & -4.96 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -4.96 & -9.66 & -4.96 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4.96 & -9.66 & -4.96 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4.96 & -9.66 & -4.96 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4.96 & -9.66 & -4.96 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4.96 & -9.66 & -4.96 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4.96 & -9.66 & -4.96 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4.96 & -9.66 & -4.96 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4.96 & -9.66 & -4.96 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4.96 & -4.83 \end{bmatrix}$$

(2.91)

Recall that the goal is to solve the problem defined in equation (2.43). Now that we have the matrix Q , it is necessary to define the right side vector $\mathbf{L}(\mathbf{f}_j)$. According to the vector described in equation (2.67), this vector will be

$$\mathbf{L}^T = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 2]. \quad (2.92)$$

Now the equation has been defined, it can easily be solved. It turns out the solution can be found using direct methods in this case by using

$$\tilde{\mathbf{x}} = Q^{-1}\mathbf{b}. \quad (2.93)$$

The following graph depicts the accuracy with which the finite element method approximates the solution to the problem at hand with uniform linear approximations.

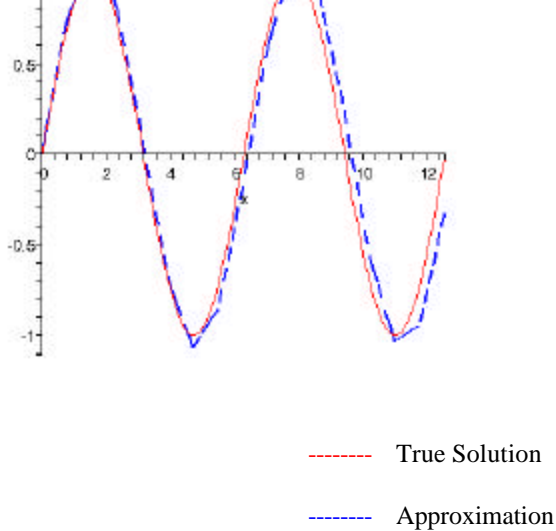


Figure 2.94 Comparative plot of the solution and approximation using uniform linear approximations to the equation $Ax=b$

The solution to the matrix equation depends on some of the parameters that were established during the finite element method. These parameters include number of partitions, the reference element chosen, and order of approximations. For example, if higher order basis functions were used, the matrix Q would have different entries than if only linear approximations were used. Thus, the solution to the equation differs depending on the order of approximations used.

Consider the problem defined in equation (2.70) with the same boundary conditions and partition; however, assume that

simply assume that the degrees of freedom exist for the problem at hand; however, they will not be specifically defined at this time. It follows that if the reference element is defined as $\hat{K} = [-1,1]$, the basis on \hat{K} can be defined as

$$\left\{ \frac{1}{2}(1 - \hat{x}), \frac{1}{2}(1 + \hat{x}), \frac{1}{2}(\hat{x}^2 - 1), \frac{1}{2}(\hat{x}^3 - \hat{x}) \right\}. \quad (2.96)$$

Assume that each K_k contains third-order approximations.

Therefore, the connectivity matrices will be 49 x 4 matrices with entries of zeroes and ones. These matrices will be used to construct M and S using $m, \hat{m}, s,$ and \hat{s} . The matrices $m, \hat{m}, s,$ and \hat{s} are found to be

$$m_i = \begin{bmatrix} \mathbf{p}/12 & \mathbf{p}/24 & -\mathbf{p}/24 & \mathbf{p}/120 \\ \mathbf{p}/24 & \mathbf{p}/12 & -\mathbf{p}/24 & -\mathbf{p}/120 \\ -\mathbf{p}/24 & -\mathbf{p}/24 & \mathbf{p}/30 & 0 \\ \mathbf{p}/120 & -\mathbf{p}/120 & 0 & \mathbf{p}/210 \end{bmatrix}, \quad (2.97)$$

$$\hat{m} = \begin{bmatrix} 2/3 & 1/3 & -1/3 & 1/15 \\ 1/3 & 2/3 & -1/3 & -1/15 \\ -1/3 & -1/3 & 4/15 & 0 \\ 1/15 & -1/15 & 0 & 4/105 \end{bmatrix}, \quad (2.98)$$

$$s_i = \begin{bmatrix} \frac{1}{p} & \frac{1}{p} & 0 & 0 \\ 0 & 0 & \frac{16}{3p} & 0 \\ 0 & 0 & 0 & \frac{16}{5p} \end{bmatrix}, \quad (2.99)$$

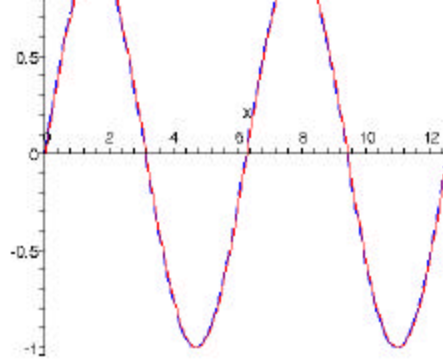
and

$$\hat{s} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & \frac{2}{5} \end{bmatrix}, \quad (2.100)$$

using equations (2.56), (2.57), (2.59), and (2.60) respectively. Thus, it follows that M and S can be constructed and used to form the matrix Q , which can then be used in the final equation

$$Q\mathbf{a} = (cM - S)\mathbf{a} = \mathbf{L}(\mathbf{f}_j). \quad (2.101)$$

The solution of this equation can be found using direct methods because the matrix Q is still a manageable size. Figure 2.102 displays both the true solution and the approximate solution to the equation when uniform third-order approximations are used.



----- True Solution

----- Approximation

Figure 2.102 Comparative plot of the solution and approximation using uniform third-order approximations to the equation $Ax=b$

Once the matrix equation has been formed, it can be solved using a variety of methods. The manner in which the matrix equation is solved can vary greatly depending on the method chosen. In the next section, a selection of methods will be explored, and the characteristics of the equation will be exploited to find the most efficient and accurate manner for solving the equation.

3. Iterative Methods

The solution to the equation $Ax = b$ can sometimes be found using direct techniques. However, these techniques are typically slow and

almost negligible error. In doing this, a solution can be found in a much more efficient and effective manner.

The following methods will be discussed as a means for solving the problem at hand: the Jacobi method, the Gauss-Seidel method, the Successive Over-Relaxation method, the Conjugate Gradient method, and the Generalized Minimum Residual method. These methods can be grouped according to the manner in which the approximations are constructed. The Jacobi, Gauss-Seidel, and Successive Over-Relaxation methods are often grouped together and called stationary or simple iterative methods. The Conjugate Gradient and the Generalized Minimum Residual methods are more complex and can be classified as Krylov Subspace methods. While the Krylov subspace methods are more sophisticated than the simple methods, they are appropriate for more of a variety of matrix problems, whereas the simple methods are quite narrow in the types of problems they can solve.

3.1. Stationary Iterative Methods

Stationary iterative methods, such as the Jacobi, Gauss-Seidel, and Successive Over-Relaxation, are methods that convert the equation $Ax = b$ into an equivalent equation of the form $x = xT + c$ for some matrix T and vector c (Burden 437). The matrix A can be split so that

$$A = M - N . \tag{3.1}$$

Therefore, the equation $Ax = b$ can be written as $(M - N)x = b$. Using simple algebra, the equation can be written as

$$T = M^{-1}N \quad (3.3)$$

and

$$c = M^{-1}b, \quad (3.4)$$

the equation can be written in the general form

$$x = xT + c. \quad (3.5)$$

Each method starts with an approximation, denoted by $x^{(0)}$, and creates consequent approximations using the equation

$$x^{(k)} = x^{(k-1)}T + c. \quad (3.6)$$

The differences between the Jacobi, Gauss-Seidel and Successive Over-Relaxation methods exist in the definitions of T and c that are formed according to the manner in which A is split.

3.1.1. Jacobi Method

The Jacobi method, the most basic iterative method, defines T and c by splitting A as

$$A = D - (L + U), \quad (3.7)$$

where D is the diagonal of A , L is the negative strictly lower-triangular part

$$x^{(k)} = D^{-1}(L+U)x^{(k-1)} + D^{-1}b. \quad (3.8)$$

Thus,

$$T = D^{-1}(L+U) \quad (3.9)$$

and

$$c = D^{-1}b. \quad (3.10)$$

Equation (3.8) can also be written as

$$x_i^{(k)} = \frac{\sum_{\substack{j=1 \\ j \neq i}}^n (-a_{ij}x_j^{(k-1)}) + b_i}{a_{ii}}, \quad i = 1, 2, \dots, n. \quad (3.11)$$

Obviously, it is necessary that $a_{ii} \neq 0$ in order to ensure that the denominator does not equal zero. It is also necessary that the matrix A be positive definite. If these conditions are satisfied, then the Jacobi method can be utilized to find an approximation to the matrix equation $Ax = b$. It also follows that if A is diagonally dominant, then the sequence of approximations produced by the Jacobi method will converge for any initial approximation (Kincaid 212). Even though the Jacobi method may find a good approximation to the solution, it may be the case that another method will find the solution faster.

method in that it uses more recent calculations in each iteration. When $i > 1$, the values $x_1^{(k)}, x_2^{(k)}, \dots, x_{i-1}^{(k)}$ have already been calculated, where $x_i^{(j)}$ is the i^{th} entry of the j^{th} approximation. Therefore, an approximation using these calculations would be more accurate than if only earlier, less-accurate calculations were used. If the components of $x^{(k)}$ are used instead of $x^{(k-1)}$ in these situations, the approximation will be closer to the true solution, provided that $x^{(k)}$ is more accurate than $x^{(k-1)}$. Thus, the Gauss-Seidel method uses the equation

$$x_i^{(k)} = \frac{-\sum_{j=1}^{i-1} (a_{ij}x_j^{(k)}) - \sum_{j=i+1}^n (-a_{ij}x_j^{(k-1)}) + b_i}{a_{ii}}, \quad i = 1, 2, \dots, n,$$

(3.12)

which leads to

$$T = (D - L)^{-1}U$$

(3.13)

and

$$c = (D - L)^{-1}b.$$

(3.14)

Again it is necessary that the matrix A be positive definite. If this condition is satisfied, then the Gauss-Seidel method can be used to approximate the solution to the equation $Ax = b$. It is also true that if A is diagonally dominant, then the sequence of approximations produced by the Gauss-Seidel method will converge for any initial approximation vector (Kincaid 216).

Clearly, if the Gauss-Seidel method converges, it is a more efficient method than the Jacobi method; however, if it does not converge, then it will diverge

3.1.3. Successive Over-Relaxation

The successive over-relaxation method, often abbreviated SOR, is another simple iterative method that requires the matrix A to be positive definite. A positive scalar value \mathbf{w} is multiplied through the equation to yield

$$\mathbf{w}(D - L - U)x = \mathbf{w}b. \quad (3.15)$$

Rearranging this equation leads to $x^{(k)} = x^{(k-1)}T_{\mathbf{w}} + c_{\mathbf{w}}$, where

$$T_{\mathbf{w}} = (D - \mathbf{w}L)^{-1}[(1 - \mathbf{w})D + \mathbf{w}U] \quad (3.16)$$

and

$$c_{\mathbf{w}} = \mathbf{w}(D - \mathbf{w}L)^{-1}b. \quad (3.17)$$

These equations can be found using simple arithmetic on equation (3.15). The SOR method will converge if the matrix A is positive definite and $0 < \mathbf{w} < 2$ (Burden 449). In fact, if the SOR method is found to converge, it may converge faster than both the Jacobi method and the Gauss-Seidel method.

It can be seen that the Gauss-Seidel method is an improvement of the Jacobi method and the SOR method is an improvement of the Gauss-Seidel. While it seems intuitive to say that each improvement will create an increase in the rate of convergence to the solution of the problem, it is helpful to look at this rate more closely to see the differences between the methods.

If the n th solution to the equation $Ax = b$ is found, $\tilde{x}^{(n)}$, then it

where $x^{(k)}$ is the k^{th} approximation. Since the true solution satisfies the equation $x = xT + c$, the error can also be defined as

$$\tilde{x} - x^{(k)} = (T\tilde{x} + c) - (Tx^{(k-1)} + c) = T(\tilde{x} - x^{(k-1)}).$$

(3.19)

It is common to measure the error using a norm and, in doing this, the following inequality is created:

$$\|\tilde{x} - x^{(k)}\| \leq \|T\| \|\tilde{x} - x^{(k-1)}\|,$$

(3.20)

which can also be written as

$$\|\tilde{x} - x^{(k)}\| \leq \|T\|^k \|\tilde{x} - x^{(0)}\|.$$

(3.21)

After taking the natural log of both sides of equation (3.21), the resulting equation,

$$\ln(\|\tilde{x} - x^{(k)}\|) = \ln(\|T\|^k \|\tilde{x} - x^{(0)}\|)$$

(3.22)

or

$$\ln(\|\tilde{x} - x^{(k)}\|) = k \ln(\|T\|) + \ln(\|\tilde{x} - x^{(0)}\|),$$

(3.23)

is a linear function of the variable $\|\tilde{x} - x^{(k)}\|$. The slope of the li

It turns out that all stationary iterative methods will converge to a solution to the equation $Ax = b$ for any initial approximation provided that $\|T\| < 1$ (Kincaid 210). This will occur if the spectral radius $\mathbf{r}(T)$, which is defined to be the maximum of the absolute value of the eigenvalues of T , is also less than 1 (Kincaid 215). In fact, the rate of convergence is the greatest when $\mathbf{r}(T)$ is minimized. If A is symmetric, then the usual 2-norm of A is equal to the spectral radius of A . The norm is often easier to work with than the spectral radius, so having this equality becomes very useful when comparing the convergence rates of the methods.

3.2. Krylov Subspace Methods

Krylov subspace methods differ from stationary methods in that the matrix and vector used to compute the approximations are dynamic in Krylov subspace methods, while the matrix and vector used in the stationary methods are fixed. These Krylov subspace methods compute approximate solutions from the Krylov subspace $K^k(A; r_o)$, where $K^k(A; r_o)$ is the k^{th} Krylov subspace defined as

$$K^k(A; r_o) = \text{span}(r_o, Ar_o, A^2r_o, \dots, A^k r_o). \quad (3.24)$$

The differences between the two Krylov subspace methods considered, the Conjugate Gradient method and the Generalized Minimum Residual method, occur in the definition of the basis for $K^k(A; r_o)$ and the manner in which the approximations are chosen.

The Conjugate Gradient method is a member of a class of methods that take the Ritz-Galerkin approach to finding approximate solutions. The approximation is found in a manner that ensures the residual is orthogonal to the Krylov subspace. That is,

On the other hand, the Generalized Minimum Residual method is a member of a class of methods that take the minimum residual approach to finding approximations. In this case, the approximations are computed to minimize the usual 2-norm over the Krylov subspace. That is,

$$\|b - Ax^{(k)}\|_2 \quad (3.26)$$

is minimized over $K^k(A; r_0)$. In what follows, a brief description of each method will occur and then both methods compared with each other both algebraically and graphically.

3.2.1 Conjugate Gradient Method

The conjugate gradient method is a useful tool in solving the matrix equation $Ax = b$. When used as a direct method, the conjugate gradient method finds the exact solution to the equation in n steps, assuming exact arithmetic, where n is the dimension of A . On the other hand, when used as an iterative method, the conjugate gradient method can approximate the solution to the equation in as few as \sqrt{n} steps (Burden 465).

Recall that the Conjugate Gradient method utilizes the Ritz-Galerkin method for finding approximate solutions to the equation at hand. Furthermore, the Conjugate Gradient method is an orthogonal projection method which seeks an approximation from the affine subspace $x_0 + K$ so that the orthogonality condition is satisfied. Because the residuals are orthogonal to each other and to the Krylov subspace, they can be considered conjugates (Saad 177).

Before going on to describe the conjugate gradient method in detail, it is necessary to discuss a preliminary theorem (Burden 465). This theorem

Theorem 3.27: The vector \tilde{x} is a solution to the positive definite linear system $Ax=b$ if and only if \tilde{x} minimizes

$$g(x) = \langle x, Ax \rangle - 2\langle x, b \rangle.$$

Proof: Assume \tilde{x} is a solution to the positive definite linear system $Ax = b$. It then remains to be shown that \tilde{x} minimizes

$$g(x) = \langle x, Ax \rangle - 2\langle x, b \rangle. \quad (3.28)$$

Consider the value of this function at some approximation, $x = x + tv$, where x and v are fixed vectors, t is some real number variable, and $v \neq 0$. Then,

$$g(x + tv) = \langle x + tv, A(x + tv) \rangle - 2\langle x + tv, b \rangle, \quad (3.29)$$

which can be simplified using the properties of inner products to get

$$g(x + tv) = \langle x, Ax \rangle + t\langle x, Av \rangle + t\langle v, Ax \rangle + t^2\langle v, Av \rangle - 2\langle x, b \rangle - 2t\langle v, b \rangle \quad (3.30).$$

Note that

$$\langle x, Av \rangle = \langle v, Ax \rangle \quad (3.31)$$

because A is symmetric. Using equation (3.31), equation (3.30) can be written as

and then simplified to

$$g(x + tv) = g(x) + 2t\langle v, Ax - b \rangle + t^2\langle v, Av \rangle. \quad (3.33)$$

Since x and v are fixed, this equation can be defined as a function of t , namely

$$h(t) = g(x + tv). \quad (3.34)$$

Because $\langle v, Av \rangle$ is positive, $h(t)$ is minimized when $h'(t) = 0$. Defining \hat{t} to be the value of t that minimizes $h(t)$, it turns out that

$$\hat{t} = \frac{-\langle v, Ax - b \rangle}{\langle v, Av \rangle} = \frac{\langle v, b - Ax \rangle}{\langle v, Av \rangle}. \quad (3.35)$$

Note that it is necessary to require that $v \neq 0$ in order to ensure that the denominator of the fraction is not zero, which would make \hat{t} undefined. Plug the value for \hat{t} back into the equation for $h(t)$ and then define $h(\hat{t}) = g(x + \hat{t}v)$. Finally,

$$g(x + \hat{t}v) = g(x) - \frac{\langle v, b - Ax \rangle^2}{\langle v, Av \rangle}, \quad (3.36)$$

$$g(x + \hat{t}v) = g(x). \quad (3.37)$$

Therefore, if \tilde{x} is the solution to $A\tilde{x} = b$, then $b - A\tilde{x} = 0$, and it follows that $g(\tilde{x} + \hat{t}v) = g(\tilde{x})$. In fact, $g(x)$ cannot be made smaller than $g(\tilde{x})$, which means that $g(x)$ is minimized by \tilde{x} .

Nevertheless, with this theorem in hand, an exploration of the conjugate gradient method can begin.

The conjugate gradient method can be used to find solutions of the equation $Ax=b$ only if the matrix A is positive definite (Burden 465). Once this qualifying characteristic has been determined, the first step in the conjugate gradient method is to make an initial approximation of the solution and to find a search direction, v , which is not equal to the zero vector. While it is unclear at the moment what is meant by “search direction,” this topic will be discussed shortly. Once an approximation has been made, the residual vector, r , can be found by

$$r = b - Ax. \quad (3.38)$$

Using the residual vector and the search direction, calculate t using the equation

$$t = \frac{\langle v, r \rangle}{\langle v, Av \rangle}, \quad (3.39)$$

where $\langle \cdot, \cdot \rangle$ is the inner-product. This t is a result of minimizing $g(x)$, by

Theorem 3.27 that $x + tv$ will give a smaller value for $g(x)$ than $x^{(0)}$.

Therefore, define a new x as $x^1 = x^0 + t_0 v^1$ and a new search direction.

Again, calculate the values for r and t and continue the process until a solution is found.

In summary, the following equations are useful in the Conjugate Gradient Method

$$r^{(k)} = b - Ax^{(k)}, \quad (3.40)$$

$$t_{(k)} = \frac{\langle v^{(k)}, b - Ax^{(k-1)} \rangle}{\langle v^{(k)}, Av^{(k)} \rangle} = \frac{\langle v^{(k)}, r^{(k-1)} \rangle}{\langle v^{(k)}, Av^{(k)} \rangle}, \quad (3.41)$$

and

$$x^{(k)} = x^{(k-1)} + t_{(k)} v^{(k)}. \quad (3.42)$$

The question now becomes how to define the search directions. There are two methods to complete this task: the method of steepest descent and the conjugate direction method.

3.2.1.1 The Method of Steepest Descent

The method of steepest descent uses the gradient to define the direction in which the greatest decrease of the $g(x)$ will occur. By breaking down $g(x)$ into its various x-components, it turns out that

(3.43)

After taking the partial derivative of $g(x)$ with respect to each $x^{(k)}$, the gradient of g will be

$$\nabla g(x) = \left(\frac{\partial g}{\partial x_1}(x), \frac{\partial g}{\partial x_2}(x), \dots, \frac{\partial g}{\partial x_n}(x) \right)^T = 2(Ax - b) = -2r. \quad (3.44)$$

Since the direction of the greatest decrease in a function is in the direction of the negative of the gradient, it follows that $g(x)$ will decrease the fastest in the direction of $-(-2r)$ or simply in the direction of r . Therefore, the method of steepest descent defines the search directions in terms of the residuals of the vectors. That is,

$$v^{(k+1)} = r^{(k)} = b - Ax^{(k)}. \quad (3.45)$$

3.2.1.2 The Conjugate Direction Method

The conjugate direction method defines the search directions as a set of nonzero, A -orthogonal vectors. That is, $V = \{v^1, v^2, \dots, v^n\}$, where $\langle v^i, Av^j \rangle = 0$ for $i \neq j$. The following theorem is necessary in this method of defining the search directions. It states

Theorem 3.46: The residual vectors $r^{(k)}$, where $k = 1, 2, \dots, n$, for a conjugate direction method, satisfy the equations $\langle r^{(k)}, v^{(j)} \rangle = 0$ for each $j = 1, 2, \dots, k$ (Burden 470).

The conjugate direction method begins with the method of steepest descent. That is, the first search direction is defined as the first residual. Assuming the initial guess is not the solution, the consequent guesses will be based on the following two equations:

$$x^{(k-1)} = x^{(k-2)} + t_{(k-1)}v^{(k-1)}, \quad (3.47)$$

where $\langle v^i, Av^j \rangle = 0$ and $\langle r^i, r^j \rangle = 0$, for $i \neq j$ and

$$v^{(k)} = r^{(k-1)} + s_{(k-1)}v^{(k-1)}. \quad (3.48)$$

The question then becomes how to define s . Since the goal is to find a set of A -orthogonal vectors, V , the s should be defined in a manner that achieves this goal. It turns out that

$$s_{(k-1)} = \frac{-\langle v^{(k-1)}, Ar^{(k-1)} \rangle}{\langle v^{(k-1)}, Av^{(k-1)} \rangle}. \quad (3.49)$$

Now that s has been defined, the A -orthogonal set of vectors can be found using equation (3.48).

The method of steepest descent and the conjugate direction method are both useful tools in defining the search directions that are necessary in the conjugate gradient method. Having chosen a $v^{(k)}$, the next step is to compute t by substituting equation (3.49) into equation (3.41) to get

(3.50)

After computing t , the r and s should also be computed. If the chosen x is not the solution, a new search direction should be found and the process repeated.

In summary, the conjugate gradient method requires the following equations:

$$r^{(0)} = b - Ax^{(0)}, \quad (3.51)$$

$$v^{(1)} = r^{(1)}, \quad (3.52)$$

and, for $k = 1, 2, 3, \dots, n$,

$$t_{(k)} = \frac{\langle r^{(k-1)}, r^{(k-1)} \rangle}{\langle v^{(k)}, Av^{(k)} \rangle}, \quad (3.53)$$

$$x^{(k)} = x^{(k-1)} + t_{(k)}v^{(k)}, \quad (3.54)$$

$$r^{(k)} = r^{(k-1)} - t_{(k)}v^{(k)}, \quad (3.55)$$

$$s_{(k)} = \frac{\langle r^{(k)}, r^{(k)} \rangle}{\langle r^{(k-1)}, r^{(k-1)} \rangle}, \quad (3.56)$$

It is helpful if the matrix A is well-conditioned to ensure proper calculations and results (Burden 456). A well-conditioned matrix is one whose condition number $K(A)$ is close to one. When A is ill-conditioned, it is possible to create a preconditioned matrix, \tilde{A} , so that an approximate solution to $Ax=b$ can be found. In this instance, the goal is to find a matrix C such that $\tilde{A} = C^{-1}AC^{-T}$. The set of equations that was used in the unconditioned matrix equation can be modified slightly to fit the new equation. The transformation for these equations from the unconditioned equations utilizes substitution and simple algebra. The preconditioned form of the Conjugate Gradient method can be used in the same manner as the unconditioned Conjugate Gradient method with the aforementioned equations as an iterative method for finding approximate solutions to $Ax = b$.

3.2.2. Generalized Minimum Residual Method

The Generalized Minimum Residual Method, often abbreviated GMRES, is an oblique projection method that seeks an approximation to the equation $Ax = b$, where the norm of the residual is minimized over the affine subspace $x^{(0)} + K^m$ for $m = 1, \dots, N$. Any vector in this subspace can be written as

$$x^{(m)} = x^{(0)} + V^m y, \tag{3.70}$$

where the column vectors of V^m form a basis of K^m and y is an m -dimensional column vector. Since the GMRES method is an oblique projection method, L_m , or the subspace of constraints, can be defined in terms of a basis W^m , where $W^m = AV^m$.

denoted $r^{(0)}$, along with the 2-norm of this residual, denoted \mathbf{D} . The first column vector, v^1 , is defined as the normalized residual, or

$$v^1 = \frac{r^{(0)}}{\mathbf{D}}. \quad (3.71)$$

The subsequent column vectors are found using an $m + 1$ by m matrix, called \overline{H}_m .

The entries of \overline{H}_m can be found using a simple algorithm. Define w_1 as the product of the matrix A and the vector v^1 . It follows that the first entry of \overline{H}_m is the inner product of v^1 and w_1 . Then w is redefined as w_1 minus this inner product. The new w is used to find the inner product of v^1 and w for the second entry in the first row of \overline{H}_m . Again, w is redefined and the process is repeated until the first row of \overline{H}_m is complete.

The process is the same for the second and subsequent rows. The entries begin on the diagonal and proceed to the right until the row is complete. The entries to the left of the diagonal will in fact be zeroes. Thus, \overline{H}_m is temporarily an upper triangular matrix. An additional entry is added to all rows, equal to the norm of the final w used to create the row. This number is placed in the entry directly below the diagonal. Once these entries have been made, \overline{H}_m is an upper Hessenberg matrix. After creating each row, a new v is created by multiplying the final w used to create the row by the inverse of the entry to the left of the diagonal.

$$x^{(m)} = x^{(0)} + V^m y_m, \quad (3.72)$$

where y_m minimizes the function

$$J(y) = \|\mathbf{b}e_1 - \overline{H}_m y\|_2. \quad (3.73)$$

It turns out that $\mathbf{b}e_1$ is a column vector defined by

$$\mathbf{b}e_1 = V_m^T(\mathbf{b}v^1). \quad (3.74)$$

Because this vector is a by-product of creating \overline{H}_m , it is only left to define the manner in which the y_k 's are found. In fact, each y_k can be found easily by computing a simple least-squares problem of dimension $(m+1) \times m$, where m is typically small (Saad 158).

3.3. Examples

A simple example can be used to illustrate each of the aforementioned iterative methods. Suppose that A is a 50×50 tridiagonal matrix with the following entries

$$\begin{bmatrix} 0 & 0 & 0 & 2.4 & 0 & 0 & 0 \\ M & 0 & 2.4 & 5 & 2.4 & 0 & M \\ 0 & 0 & 0 & 2.4 & 0 & 0 & 0 \\ M & 0 & 0 & 0 & 0 & 5 & 2.4 \\ 0 & \Lambda & 0 & \Lambda & 0 & 2.4 & 3 \end{bmatrix}, \quad (3.75)$$

and suppose that the right side of the equation b is a 50×1 vector of the form

$$[1 \ \Lambda \ 1 \ \Lambda \ 1 \ \Lambda \ 1 \ \Lambda \ 1]^T. \quad (3.76)$$

Because this example is quite small, the true solution to the equation can be found using direct techniques. After finding the inverse of A , the true solution can be defined as

$$x = A^{-1}b. \quad (3.77)$$

With the solution in mind, each iterative technique can be used to find an approximation, and then the approximate solutions can be compared to the true solution to determine the accuracy of the methods. Finally, the five methods can be compared with each other to find the method that converges the fastest.

Because the matrix A is positive definite, the Jacobi method for approximating a solution can be used. The following graph shows the manner in which the Jacobi method approximates the true solution to the equation.

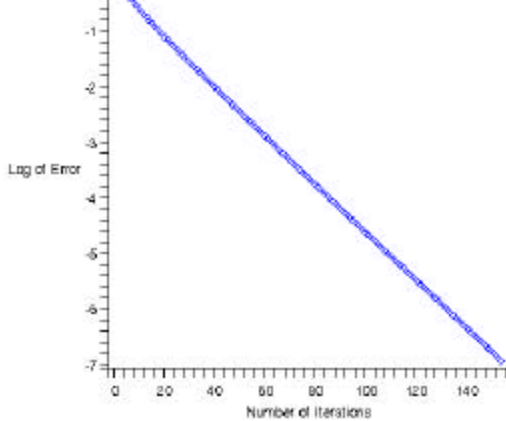


Figure 3.78 Convergence Rate of Jacobi Method

Clearly, the Jacobi method approximates the solution to a good degree of accuracy. However, note that it takes about 150 iterations to achieve an error of 10^{-7} . When the Gauss-Seidel method is used to find an approximate solution, it takes only 70 iterations to reach an error of 10^{-7} . The graph below shows the true solution and the Gauss-Seidel approximation.

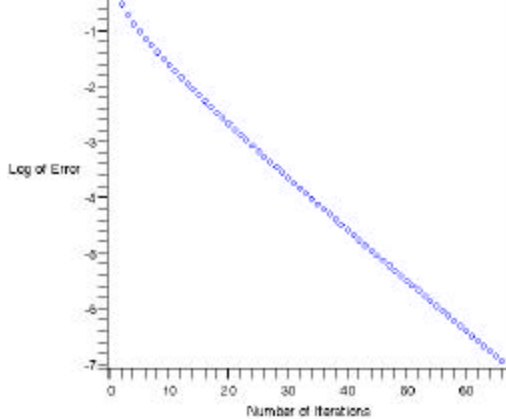


Figure 3.79 Convergence Rate of Gauss-Seidel Method

The Successive Over-Relaxation turns out to converge the fastest of the three stationary methods tested in this example. The following graph shows the manner in which two approximations converge to a solution found using the SOR method.

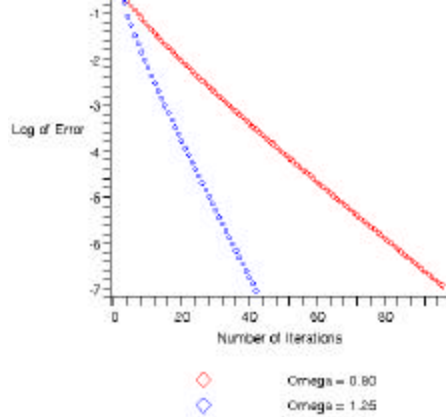


Figure 3.80 Convergence Rate of Successive Over-Relaxation Method

Each approximation uses a different value for the scalar quantity ω , which is used in computing the iterations in the method. The choice of ω is an important factor in the convergence rate of the SOR method. In fact, recall that the method will only converge for a range of values of ω between 0 and 2. In this example, it turns out that the method converged faster when ω was chosen to be 1.25.

The Krylov subspace methods, namely the Conjugate Gradient method and the Generalized Minimum Residual method, converge much faster than any of the aforementioned stationary methods. The Conjugate Gradient method converges to an approximate solution with an error of 10^{-7} in about 25 iterations. The following graph shows the true solution and the approximation for the Conjugate Gradient method.

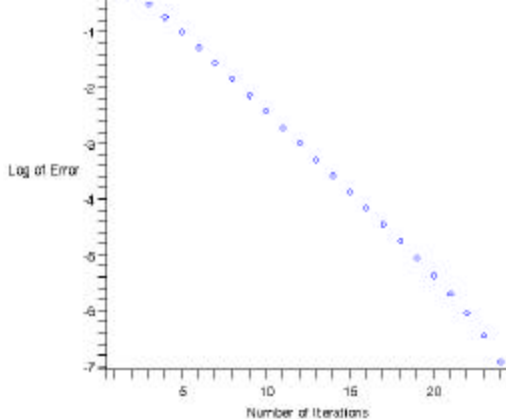


Figure 3.80 Convergence Rate of Conjugate Gradient Method

The algorithm for the Generalized Minimum Residual method is difficult to follow and even more difficult to debug. Most of the trouble lies with the creation of the orthonormal basis V^m . It is difficult to look at the algorithm and understand the action taken at each step, making it hard to write a program to carry these actions out. Because of the troubles encountered in writing the program to test the Generalized Minimum Residual method, the program was never debugged completely and would be the first area of attention in the future.

Nevertheless, it is clear that each of the four methods approximates the true solution to the equation with quite a degree of accuracy. However, the rate at which the methods converge differs greatly as each one is used. The following table displays the convergence rates for the four iterative methods.

| Method | Number of | Error |
|--------|-----------|-------|
| | | |

| | | |
|--------------------|-----|-----------|
| Gauss-Seidel | 70 | 10^{-7} |
| SOR – Omega = 0.80 | 100 | 10^{-7} |
| SOR – Omega = 1.25 | 45 | 10^{-7} |
| Conjugate Gradient | 25 | 10^{-7} |

Table 3.82 Comparative Chart of Convergence Rates

Obviously, the Conjugate Gradient method converges the fastest and to the smallest degree of error. This method would be the ideal choice for approximating the solution to the problem at hand.

4. Solving the Helmholtz Equation

Now that detailed descriptions of the finite element method and the five iterative methods have been provided, it is time to use these iterative methods to solve the matrix equation that results from the finite element method in an attempt to solve the one-dimensional Helmholtz equation. Recall that the finite element method was used to approximate a solution to the partial differential equation, the Helmholtz equation. The result of this method is a matrix equation $Q\mathbf{a} = \mathbf{L}$, where Q will be an extremely large matrix when a real-world problem is being modeled. Therefore, the five

solve and allow for the usage of any of the five iterative methods discussed. On the other hand, Q may not always be positive definite or strictly diagonally dominant. The interested reader can find a proof of this characteristic in the appendix. The matrix Q is usually not positive definite. Because this characteristic plays an important role in many of the methods, the matrix can be conditioned so that this quality is fulfilled. A new Q can be defined by multiplying the original matrix Q and its transpose. That is,

$$Q = Q^T Q . \tag{4.1}$$

Even though the matrix Q is now positive definite, it is difficult to explicitly state whether or not the spectral radius is less than one. Because this trait is necessary for the convergence of the Jacobi method and the Gauss-Seidel method, it is impossible to say whether or not these methods will converge to a solution to the equation. If these methods do converge to a good approximation, it will be done so in an undesirable manner. Therefore, it is unlikely that the Jacobi method and the Gauss-Seidel method would be used to find a solution to the matrix equation resulting from the finite element method.

In the SOR method, the Conjugate Gradient method, and the Generalized Minimum Residual method, it is more common for the iterations to converge to a solution with negligible error in a desirable manner. Because the Krylov subspace methods are more efficient than the stationary methods, the Conjugate Gradient and Generalized Minimum Residual methods should be considered superior to the SOR method. The two Krylov subspace methods should be used to solve matrix equations before trying to use the SOR method. In fact, because these methods will converge for a wider array of problems and will do so in a faster manner, they should almost always be the first choice to solve the equation at hand.

Recall that the goal is to find the solution to the one-dimensional Helmholtz equation in the most efficient manner possible. Because this

Generalized Minimum Residual method are efficient ways to solve the matrix equation that results from the finite element method. The steps outlined in this thesis will lead to an approximate solution to the Helmholtz equation in a quick and economic manner.

5. Further Considerations

This thesis explored many of the important aspects of the finite element method and selected iterative methods for solving the one-dimensional Helmholtz equation. However, there was not enough time to explore each of the fascinating areas that arose. As questions were answered, more questions appeared, creating an infinite cycle of “ask and answer.” However, many of these questions are still left unanswered and would be interesting to look into in the future.

There are quite a few areas within the finite element method that would be interesting to consider further. For example, one of the first steps in the method is to partition the domain. Changes in the size of the subintervals created and variations in the uniformity of the partition may alter the accuracy or efficiency of the method. These are two areas that would be fascinating to look into further in an attempt to find the best way to find a solution to the Helmholtz equation. Similarly, the choice of basis functions and the reference element affects the manner in which the finite element method works. It would be exciting to explore the various choices of basis functions and reference elements and the effects they have on the accuracy and efficiency of the method in finding an approximation.

Once the basis functions have been chosen, it would be interesting to consider using these functions in both uniform and non-uniform approaches. This thesis only considered uniform approximations over the domain; however, there is nothing preventing the usage of linear approximations in one subinterval and eighth order approximations in another. These changes could be examined for the effects on the precision of the approximation as well as the efficiency.

methods. Clearly, some of the methods considered in this thesis would not be very useful when applying them to real-world problems. Nevertheless, some of the methods, more specifically the Conjugate Gradient and Generalized Minimum Residual methods, could be used to solve many of these problems. Preconditioners can be used in each of these methods to speed up the rates of convergence. It would be interesting to study these preconditioners further and look into the effects that they have on the convergence rates and accuracy.

Finally, the programs created for studying this thesis were done so in a program called *Maple*. One of the most applicable changes that could be made in studying the finite element method and all of the iterative methods would be in the manner in which they are programmed and used to test the underlying theory. Other programming environments, including *MatLab*, *Fortran*, and *C++*, could also have been used to program these tests and may actually have been better choices for doing so. It would be very interesting and helpful to experiment with some of these environments and determine the best one to use for study in the relevant areas.

6.1 Is Q strictly diagonally dominant?

It has been shown that $c(u, v) - (u', v') = \mathbf{L}(v)$ is the variational problem, derived from the differential equation $u'' + cu = 0$. In the discrete problem, the left side corresponds to Q, where $Q = cM - S$. At times it is important that Q is strictly diagonally dominant. The $n \times n$ matrix Q is defined as thus if and only if

$$|Q_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^n |Q_{ij}|, \quad 1 \leq i \leq n. \quad (6.1)$$

The matrix Q can be understood more easily if it is broken down into its components. Therefore, it is useful to first look at M and S individually. When higher order approximations are used, the linear approximation matrix will be a submatrix of the higher order approximation matrix. Therefore, if the linear approximation matrix is not strictly diagonally dominant, then, similarly, the higher-order matrix will not be strictly diagonally dominant. For the case of uniform linear approximations, recall that each entry in the matrix M is defined as

$$M_{ij} = \begin{cases} \int_{K_{i-1}} \mathbf{f}_i^2(x) dx + \int_{K_i} \mathbf{f}_i^2(x) dx, & i = j \\ \int_{K_{\min(i,j)}} \mathbf{f}_i(x) \mathbf{f}_j(x) dx, & i \neq j \end{cases}. \quad (6.2)$$

$$M_{ij} = \begin{cases} \frac{h_{\min(i,j)}}{2} \int_{\hat{K}} \hat{\mathbf{f}}_0(\hat{x}) \hat{\mathbf{f}}_1(\hat{x}) d\hat{x}, & i \neq j \end{cases},$$

(6.3)

where

$$\hat{\mathbf{f}}_0(\hat{x}) = \frac{1}{2}(1 - \hat{x})$$

(6.4)

and

$$\hat{\mathbf{f}}_1(\hat{x}) = \frac{1}{2}(1 + \hat{x}).$$

(6.5)

Similarly, recall that the entries of the matrix S can be defined as

$$S_{ij} = \begin{cases} \int_{K_{i-1}} (\mathbf{f}'_i)^2(x) dx + \int_{K_i} (\mathbf{f}'_i)^2(x) dx, & i = j \\ \int_{K_{\min(i,j)}} \mathbf{f}'_i(x) \mathbf{f}'_j(x) dx, & i \neq j \end{cases}.$$

(6.6)

Using a change of variable again, the entries can also be written as

$$S_{ij} = \begin{cases} \frac{h_{\min(i,j)}}{2} \int_{\hat{f}} \hat{f}'_0(\hat{x}) \hat{f}'_1(\hat{x}) d\hat{x}, & i \neq j \end{cases} \quad (6.7)$$

The first and last rows of the simple matrix are very similar in construction. Therefore, the construction of these rows can be examined to determine the presence or absence of strict diagonal dominance. It turns out that these rows allow for strict diagonal dominance if $|4c - 3| > |2c + 3|$. Therefore, values of c less than zero create the opportunity for Q to be strictly diagonally dominant. The middle rows have a similar construction as well. If $|2c + 3| < |4c - 3|$ then the middle rows do not prevent strict diagonal dominance from occurring. This occurs only when c is less than zero. Therefore, the matrix will be strictly diagonally dominant if c is less than zero.

The practical application of the Helmholtz equation limits what the value of c could be. It turns out that $c = k^2$. The right side of this equation will always be a number greater than or equal to zero. Therefore, the left side of the equation must also be a number greater than or equal to zero. With this in mind, a negative value of c , which would make the matrix Q strictly diagonally dominant, is impossible. Therefore, the matrix Q will never be strictly diagonally dominant when linear approximations are used.

Linear approximations are the simplest form of approximation. Thus, a higher order approximation will only build upon the results of the linear approximations. Since it has been shown that linear approximations do not allow for a strictly diagonally dominant matrix Q , it is also impossible for higher order approximations to have the desired trait.

Although the matrix Q is not strictly diagonally dominant, it is necessary to check if it is diagonally dominant. That is, Q is diagonally

The matrix Q also fails to be diagonally dominant, which can easily be proven in a similar manner as above. This characteristic of Q is very useful at times when choosing an appropriate method for solving problems.

6.2. Finite Element Method Program

```
lensub:=proc(p)
local j,i;
for i from 1 to nops(p)-1 do
j[i]:=p[i+1]-p[i];
od;
convert(convert(j,list),Vector);
end proc;
```

```
Nsize:=proc(deg)
local i,d,n;
d:=convert(deg,Vector);
n:=0;
for i from 1 to Dimension(d) do
n:=n+d[i];
od;
n:=n+1;
end proc;
```

```
maxdeg:=proc(deg)
sort(deg)[-1];
end proc;
```

```
Lam:=proc(N,mx)
```



```

LMatrix[i]:=L:
od:

ct:=nops(p):
f:Matrix(nops(p)-1,mx-1):
for k from 1 to nops(p)-1 do
for m from 1 to deg[k]-1 do
f[k,m]:=ct+1:
ct:=ct+1:
od:
od:
for r from 1 to nops(p)-1 do
LM:=LMatrix[r];
for s from 1 to deg[r]-1 do
LM[f[r,s],s+2]:=1:
od:
LMatrix[r]:=LM;
od:
convert(LMatrix,list):
end proc:

```

```

LamTrans:=proc(L)
local i,LT;
for i from 1 to nops(L) do
LT[i]:=Transpose(L[i]);
od:
convert(LT,list);
end proc:

```

```

lfunc:=proc(mx,l0,l1)
local i,j,l;
l[0]:=l0;
l[1]:=l1;
for j from 2 to mx do
l[j]:=int(P(i-1 x) x=-1..z):

```

```
local i,j,mht:
mht:=Matrix(mx+1,mx+1):
for i from 1 to mx+1 do
for j from 1 to mx+1 do
mht[i,j]:=int(lf[i]*lf[j],z=-1..1):
od:
od:
mht;
end proc:
```

```
shat:=proc(mx)
local i,j,sht:
sht:=Matrix(mx+1,mx+1):
for i from 1 to mx+1 do
for j from 1 to mx+1 do
sht[i,j]:=int(diff(lf[i],z)*diff(lf[j],z),z=-1..1):
od:
od:
sht;
end proc:
```

```
subinttimesmhat:=proc(vc,m,h)
local i,mi:
for i from 1 to Dimension(h) do
mi[i]:=vc[i]^2*(h[i]/2)*m;
#print([i,vc[i]]);
od:
convert(mi,list);
end proc:
```

```
subinttimesshat:=proc(s,h)
local i,si:
for i from 1 to Dimension(h) do
si[i]:=(2/h[i])*s;
od:
```

```

M:=Matrix(N,N):
for i from 1 to nops(L) do
M1[i]:=((L[i].sitmh[i]).LT[i]):
od:
M2:=convert(M1,list):
for j from 1 to nops(M2) do
M:=Add(M,M2[j]):
od:
M;
end proc:

```

```

SMatrix:=proc(sitsh)
local S,i,j,S1,S2;
S:=Matrix(N,N);
for i from 1 to nops(L) do
S1[i]:=((L[i].sitsh[i]).LT[i]):
od:
S2:=convert(S1,list):
for j from 1 to nops(S2) do
S:=Add(S,S2[j]):
od:
S;
end proc:

```

```

QMatrix:=proc(M,S)
local QMat:
QMat:=M-S;
end proc:

```

```

Rvector:=proc(va,vb,NN,np)
local f,r,rvec:
for f from 1 to NN do
r[f]:=0;
od:
rvec:=convert(convert(r,list),Vector):

```

end proc:

Np:=16:

a:=0:

b:=4*Pi:

```
for i from 0 to Np do
ptemp[i+1]:=a + (i)*(b-a)/Np;
od:
```

p:=convert(ptemp,list):

c:=1;

```
for i from 1 to Np do
degtemp[i]:=1;
vc[i]:=c;
od:
```

deg:=convert(degtemp,list);

subint:=(nops(p))-1:

endpts:=[p[1],p[-1]]:

l0:=(1/2)*(1-z):

l1:=(1/2)*(1+z):

u:=x->sin(c*x);

h:=lensub(p):

N:=Nsize(deg):

mx:=maxdeg(deg):

L:=Lam(N,mx):

L T:=LamTrans(L):

M:=MMatrix(sith,L,LT):

S:=SMatrix(sitsh):

Q:=evalf(QMatrix(M,S)):

for i from 1 to N do

Q[i,1]:=0:

Q[1,i]:=0:

od:

Q[1,1]:=1:

vala:=c*cos(c*a):

valb:=-c*cos(c*b):

R:=Rvector(vala,valb,N,Np+1):

R[1]:=0:

Qinv:=MatrixInverse(Q):

alpha:=MatrixVectorMultiply(Qinv,R):

for it from 1 to Np do

coefs:=MatrixVectorMultiply(Transpose(L[it]),alpha);

zz:=2*x/(p[it+1]-p[it]) - (p[it+1]+p[it])/(p[it+1]-p[it]):

temp:=0;

for dg from 1 to deg[it]+1 do

temp:=temp+coefs[dg]*subs(z=zz,lf[dg]);

od:

temp[it]:=temp*(Heaviside(x-p[it])-Heaviside(x-p[it+1]));

od:

```
for i from 1 to N do
  for j from 1 while j<i do
    L_A[i,j]:=-A[i,j];
  od:
od:
```

```
U_A:=Matrix(N,N);
for j from 1 to N do
  for i from 1 while j>i do
    U_A[i,j]:=-A[i,j];
  od:
od:
```

```
D_A:=Matrix(N,N);
for i from 1 to N do
  D_A[i,i]:=A[i,i];
od:
```

```
T_J:=MatrixMatrixMultiply(MatrixInverse(D_A),MatrixAdd(L_A,U_A));
```

```
c_J:=MatrixVectorMultiply(MatrixInverse(D_A),R);
```

```
x_sol:=MatrixVectorMultiply(MatrixInverse(A),R);
```

```
xJ[0]:=Vector(N);
```

```
tol:=0.001;
```

```
errJ:=Norm((VectorAdd(x_sol,-(xJ[0])),2)/(Norm(x_sol,2)):
ErrorPJ[0]:=Norm((VectorAdd(x_sol,-(xJ[0])),2)/(Norm(x_sol,2)):
for i from 1 while errJ > tol do
  xJ[i]:=evalf(VectorAdd(MatrixVectorMultiply(T_J,xJ[i-1]),c_J)):
  errJ:=evalf(Norm((VectorAdd(x_sol,-(xJ[i-1])),2)/(Norm(x_sol,2))));
  ErrorPJ[i]:=errJ:
od:
```

```

for i from 1 to N do
  for j from 1 while j<i do
    L_A[i,j]:=-A[i,j];
  od:
od:
U_A:=Matrix(N,N):
for j from 1 to N do
  for i from 1 while j>i do
    U_A[i,j]:=-A[i,j];
  od:
D_A:=Matrix(N,N):
for i from 1 to N do
  D_A[i,i]:=A[i,i];
od:

T_G:=MatrixMatrixMultiply(MatrixInverse(MatrixAdd(D_A,-
(L_A))),U_A):

c_G:=MatrixVectorMultiply(MatrixInverse(MatrixAdd(D_A,-(L_A))),R):

x_sol:=MatrixVectorMultiply(MatrixInverse(A),R):

xG[0]:=Vector(N):

tol:=0.001:

errG:=Norm((VectorAdd(x_sol,-(xG[0]))),2)/(Norm(x_sol,2)):
ErrorPG[0]:=Norm((VectorAdd(x_sol,-(xG[0]))),2)/(Norm(x_sol,2)):
for i from 1 while errG > tol do
xG[i]:=evalf(VectorAdd(MatrixVectorMultiply(T_G,xG[i-1]),c_G)):
errG:=evalf(Norm((VectorAdd(x_sol,-(xG[i-1]))),2)/(Norm(x_sol,2)));
ErrorPG[i]:=errG:
EG[i]:=i,log(ErrorPG[i]):
od:

```

```
for i from 1 to N do
  for j from 1 while j<i do
    L_A[i,j]:=-A[i,j];
  od:
od:
```

```
U_A:=Matrix(N,N);
for j from 1 to N do
  for i from 1 while j>i do
    U_A[i,j]:=-A[i,j];
  od:
od:
```

```
D_A:=Matrix(N,N);
for i from 1 to N do
  D_A[i,i]:=A[i,i];
od:
```

w:=1.25:

```
T_S:=MatrixMatrixMultiply(MatrixInverse(MatrixAdd(D_A,MatrixScalarM
ultiply(-(L_A),w))),MatrixAdd(MatrixScalarMultiply(D_A,(1-
w)),MatrixScalarMultiply(U_A,w)));
c_S:=MatrixVectorMultiply(MatrixScalarMultiply(MatrixInverse(MatrixAdd
(D_A,MatrixScalarMultiply(-(L_A),w))),w),R):
```

```
x_sol:=MatrixVectorMultiply(MatrixInverse(A),R):
```

```
xS[0]:=Vector(N):
```

```
for k from 1 to Dimension(xS[0]) do
  xS[0][k]:=1:
od:
```



```

xS[i]:=evalf(VectorAdd(VectorAdd(T_S,xS[i-1]),c_S)):
errS:=evalf(Norm((VectorAdd(x_sol,-(xS[i-1]))),2)/(Norm(x_sol,2)));
ErrorPS[i]:=errS:
ES[i]:=i,log(ErrorPS[i]):
od:

```

6.6. Conjugate Gradient Method Program

```

xC[0]:=Vector(N):

C:=Matrix(N,N):
for j from 1 to N do
C[j,j]:=1:
od:

tol:=0.001:

rC[0]:=VectorAdd(R,-(MatrixVectorMultiply(A,xC[0]])):
wC[0]:=MatrixVectorMultiply(C,rC[0]):
vC[1]:=MatrixVectorMultiply(C,wC[0]):

a:=0:
for i from 1 to Dimension(wC[0]) do
a:=a+(wC[0][i]*wC[0][i]):
od:

errC:=Norm((VectorAdd(x_sol,-(xC[0]])),2)/(Norm(x_sol,2)):
ErrorPC[0]:=Norm((VectorAdd(x_sol,-(xC[0]])),2)/(Norm(x_sol,2)):

for i from 1 while errC > tol do

uC[i]:=MatrixVectorMultiply(A,vC[i]):

tb:=0:
for k from 1 to Dimension(uC[i]) do

```

```
wC[i]:=MatrixVectorMultiply(C,rC[i]):
```

```
B:=0:
```

```
for m from 1 to Dimension(wC[i]) do
```

```
B:=evalf(B+(wC[i][m]*wC[i][m])):
```

```
od:
```

```
sC[i]:=evalf(B/a):
```

```
vC[i+1]:=VectorAdd(MatrixVectorMultiply(C,wC[i]),VectorScalarMultiply(  
vC[i],sC[i]));
```

```
a:=B:
```

```
errC:=evalf(Norm((VectorAdd(x_sol,-(xC[i-1]))),2)/(Norm(x_sol,2)));
```

```
ErrorPC[i]:=errC:
```

```
EC[i]:=i,log(ErrorPC[i]):
```

```
od:
```

absolute value:

The absolute value function is defined as

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x \leq 0 \end{cases} \text{ and is denoted}$$
$$f(x) = |x|.$$

affine transformation:

An affine transformation is a transformation that preserves collinearity and ratios of distances.

algorithm:

An algorithm is a set of instructions for carrying out a procedure.

A-orthogonal:

A set of vectors v_i is said to be A -orthogonal if

$$\langle v_i, Av_j \rangle = 0 \text{ for } i \neq j.$$

basis:

A sequence of vectors forms a basis for a subspace V if they are linearly-independent and they span V .

boundary value problem:

A boundary value problem is a problem in which the boundary conditions are

used to find the solution.

Characteristic Polynomial:

The characteristic polynomial is created in an attempt to find the eigenvalues of a matrix A . In searching for these values, the equation $\det(A - \lambda I) = 0$ must be solved. This equation is called the characteristic polynomial for the matrix A .

Column vector:

A column vector, also called a *vector*, is a matrix with only one column.

$\kappa(A) = \frac{\|A\|}{\|A^{-1}\|}$, where $\|\cdot\|$ is some natural matrix norm.

Continuous: A function is continuous at $x = c$ if the function is defined at c and $\lim_{x \rightarrow c} f(x) = f(c)$.

Converse: The converse of $P \Rightarrow Q$ is $Q \Rightarrow P$.

Cross Product: The cross product of two vectors, u and v , is a vector perpendicular to both u and v . The magnitude of this vector is the area of the parallelogram formed by the two vectors u and v .

Denominator: The denominator of the fraction $\frac{p}{q}$ is q .

Diagonally dominant: A matrix A is said to be diagonally dominant if

$$|a_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^n |a_{ij}| \text{ for } 1 \leq i \leq n.$$

Differential Equation: A differential equation is an equation that gives information about the rate of change of an unknown function.

Domain: The domain of a function is the set of all input numbers of that function.

Dot Product: see *inner product*

Eigenvalue: An eigenvalue of a square matrix A is a scalar

Error: The error is defined as the absolute value of the difference between an estimated value and the true value being estimated.

Finite Element Method: The finite element method is a method for solving an equation by approximating continuous quantities as a set of quantities at discrete points.

Function: A function f between nonempty subsets A and B is a relation such that $f \subseteq A \times B$ and if $(a, b) \in f$ and $(a, b') \in f$, then $b = b'$.

Function space: A function space is a collection of real-valued continuous functions defined on an interval.

Gradient: The gradient vector of a differentiable function f at the point (a, b) is defined as
$$\text{grad } f(a, b) = f_x(a, b)\mathbf{i} + f_y(a, b)\mathbf{j}.$$

Helmholtz Equation: The Helmholtz equation is an elliptic partial differential equation of the form
$$\Delta u(x) + k^2 u(x) = 0 \text{ for } x \in \mathbf{R}^n.$$

Hermitian: A matrix A is said to be Hermitian if $A^* = A$, where A^* is the conjugate transpose of A .

Ill-conditioned: A matrix A is said to be ill-conditioned if its condition number is significantly greater than one.

Inner product: The inner product, also called *dot product*, of two

product of two vectors is a scalar quantity.

Integration by parts: A function can be integrated by parts using the formula $\int uv'dx = uv - \int u'vdx$.

Interval: A range of numbers is an interval. A close interval $[a, b]$ satisfies $\{x \mid a \leq x \leq b\}$. An open interval (a, b) satisfies $\{x \mid a < x < b\}$. A half-open (or half-closed) interval is the range of numbers between a and b including either a or b but not both.

Inverse: The inverse of a matrix A , denoted A^{-1} , satisfies the equation $A^{-1}A = I$, where I is the identity matrix.

Iterative method: An iterative method is a technique for approximating a solution to the linear system $Ax = b$ by generating a sequence of vectors that converges to the solution.

Krylov subspace: The k th Krylov subspace is defined as $K^k(A; r_0) = \text{span}(r_0, Ar_0, A^2r_0, \dots, A^k r_0)$.

Krylov subspace method: A Krylov subspace method seeks an approximate solution to a problem from the Krylov subspace $K^k(A; r_0)$.

$$\|x\|_2 = \left\{ \sum_{i=1}^n x_i^2 \right\}^{1/2}.$$

Least squares problem: The least squares approach to solving a problem determines the best approximate solution to a problem when the error is defined as the sum of the squares of the differences between the values of the approximation and the values of the true solution to the problem.

Legendre polynomial: The Legendre polynomials are solutions to the Legendre differential equation and are denoted $P_n(x)$.

Linear combination: A vector b is called a linear combination of vectors v_i if there are scalars x_i such that

$$b = \sum_i x_i v_i.$$

Linearly-independent: A sequence of vectors is said to be linearly-independent if none of them can be written as a linear combination of the others.

Lower Triangular: A matrix A is said to be lower triangular if

$$a_{ij} = 0 \text{ for } i < j.$$

Mapping: A mapping, also called a *transformation* or *function*, from \mathbf{R}^n to \mathbf{R}^m assigns each vector x in \mathbf{R}^n , the domain, to a vector $T(x)$ in \mathbf{R}^m , the range.

Minimum residual approach: The minimum residual approach applies to a class of Krylov subspace methods which uses the characteristic that $\|b - Ax\|_2$ is minimized over the current Krylov subspace.

Natural logarithm: The natural logarithm, also called the *natural log*, of x is the power of e needed to get x and is denoted $\ln x = c$, where c is the natural log of x . It can also be looked at as $e^c = x$.

Natural norm: The natural, or induced, matrix norm is defined as
$$\|A\| = \max_{\|x\|=1} \|Ax\|.$$

Nonsingular: A matrix is said to be nonsingular if it is an invertible matrix. That is, the inverse of the matrix exists and can be found.

Numerator: The numerator of the fraction $\frac{p}{q}$ is p .

Orthogonal: Two vectors are orthogonal to each other if their inner product equals zero.

Orthonormal: A set of vectors is said to be orthonormal if the vectors are orthogonal unit vectors.

Partial differential equation: A partial differential equation is an equation involving functions and their derivatives.

Partition: A partition is a collection of nonempty subsets such that every point in the set belongs to one

Polynomial: A polynomial of degree n is
$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$
 where
the a_i 's are constant coefficients and $a_n \neq 0$.

Positive definite: A matrix A is positive definite if $x^T A x > 0$ for every nonzero vector x . The matrix is also positive definite if $\langle Ax, x \rangle > 0$ for all $x \neq 0$.

Relation: A relation between two sets is any subset of the Cartesian product of these sets.

Residual vector: The residual vector is defined as $r = b - Ax$ where x is an approximation to the solution of the equation $Ax = b$.

Ritz-Galerkin approach: The Ritz-Galerkin approach is a class of Krylov subspace methods which requires that the residual is orthogonal to the current Krylov subspace. That is, $r^k \perp K^k(A; r_0)$.

Row vector: An n -dimensional row vector is a $1 \times n$ matrix.

Sequence: A sequence is a function whose domain is the natural numbers.

Slope: The slope of a line $y = mx + b$ is m , the rate of change of y with respect to x .

Span: The span of a set of vectors is the set of all linear combinations of those vectors.

Stationary methods: Stationary methods are iterative methods for solving $Ax = b$ in which the approximations are expressed in the form $x^{(k)} = Tx^{(k-1)} + c$, where T is a fixed matrix and c is a fixed vector.

Strictly diagonally dominant: A matrix A is said to be diagonally dominant if $|a_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^n |a_{ij}|$ for $1 \leq i \leq n$.

Strictly upper triangular: A matrix A is said to be strictly upper triangular if $a_{ij} = 0$ for $i \geq j$.

Strictly lower triangular: A matrix A is said to be strictly lower triangular if $a_{ij} = 0$ for $i \leq j$.

Subspace: A subspace of \mathbf{R}^n is a subset of \mathbf{R}^n that contains the zero vector, is closed under addition and is closed under scalar multiplication (Bretscher 110).

Symmetric: A square matrix A is said to be symmetric if $A^T = A$, where A^T is the transpose of A .

Transpose: The transpose of an $n \times m$ matrix A , denoted A^T , is the matrix where the entry in the i^{th} row and j^{th} column is the entry from the j^{th} row and i^{th} column of A .

Tridiagonal: A matrix A is said to be tridiagonal if $a_{ij} = 0$ for

dt^2 dt **Upper triangular:**

A matrix A is said to be upper triangular if $a_{ij} = 0$ for $i > j$.

Vector space:

A vector space V is a set of vectors for which addition and scalar multiplication are defined and the following ten axioms hold true for all vectors u, v , and w in the space and scalars c and d .

1. $u + v \in V$
2. $u + v = v + u$
3. $(u + v) + w = u + (v + w)$
4. The zero vector exists such that $u + 0 = u$.
5. An additive inverse exists such that $u + (-u) = 0$.
6. $c \cdot u \in V$
7. $c \cdot (u + v) = c \cdot u + c \cdot v$
8. $(c + d) \cdot u = c \cdot u + d \cdot u$
9. $c \cdot (d \cdot u) = (c \cdot d) \cdot u$
10. A multiplicative identity exists such that

condition number is close to one.

Zero vector: A zero vector is a vector whose entries are all zero.

8. Definition of Symbols

\prod product of

\sum sum of

\int integral

\mathbf{I} intersection

\mathbf{Y} union

(\cdot, \cdot) inner product

\mathbf{d}_{ij} delta function

$\| \cdot \|_2$ 2-norm

\perp orthogonal

\mathbf{R} set of real numbers

\overline{D} closure of the set D

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PREDICTORS OF COLLEGE ADJUSTMENT

Helene J. Mizrahi

Abstract

The first phase of a longitudinal study of psychosocial predictors of college adjustment is reported here. During data collection one, 141 freshmen (22.7% males, 77.3% females) completed the GEIS (emotional intelligence), NEO-PIR conscientiousness scale (CS), BDI (depression) AMS (motivation) and two open ended questions on coping with anger and sadness to tap resiliency, and a demographic questionnaire. It was hypothesized that depression would be negatively correlated with the other psychosocial variables and that EQ, conscientiousness, motivation, and resiliency would be positively intercorrelated. Depression was negatively correlated with AMS, CS, and GEIS (all $p < .001$). Also, emotional intelligence, motivation, and conscientiousness were all positively intercorrelated (all $p < .001$). Those who reported resilient coping with experiences of anger were higher on GEIS ($p < .01$), and those who reported resilient coping with sadness were higher on AMS, CS, and GEIS, all ($p < .05$). During data collection two these predictors were regressed on students' adjustment to college scores (SACQ) and during data collection three they were regressed on students' GPA's in Spring semester.

some students thrive on the opportunities college presents, others wither under the stress from the demands of college life (Leong & Bonz, 1997). If we can identify individuals whose personal traits and characteristics pose a threat for positive college adjustment, they could benefit from early intervention strategies thereby lessening the consequences of poor adjustment.

High school GPA and Scholastic Assessment Tests (SAT) scores account for approximately 25% of the variance in college GPA (Wolfe & Johnson, 1995; Tross, Harper, Osher, & Kneidinger, 2000). Although gender, race, and socioeconomic factors have been shown to affect the adjustment process, the present study focuses on other variables to account for the remaining 75% of the variance in college performance. Some of these domains include: personality factors (conscientiousness), parent-child relationships (specifically attachment styles), emotional intelligence, coping styles and resiliency, and depression, which all play a pivotal role in the adjustment process. These variables are explored in an attempt to isolate specific factors which hinder an optimum college adjustment, and to identify individuals that might be at risk, so they can receive early intervention.

College adjustment is especially crucial for freshmen students (Bragg, 1994), who are experiencing a new environment and feelings of anonymity. In addition, those students who begin college with unrealistic expectations can suffer from maladjustment and in some cases leave a college permanently (Pancer & Hunsberger, 2001).

The effects of parenting styles on college adjustment have been researched extensively (e.g. Wintre & Yaffe, 2000; Strage & Brandt, 1999), and the findings have shown that an authoritative parenting style is more advantageous for children than authoritarian or permissive parenting styles. Some of the advantages include students earning better grades, a greater degree of intrinsic motivation in regard to school success, and the development of an internal locus of control over scholastic endeavors. Attachment theorists suggest that children develop an "Internal working model" (Huff, 2001) consisting of the child, the parent or attachment figure, and the relationship between the parent and child. This internal model is

college adjustment was. Wintre and Yaffe (2000) based their research on a model linking parenting styles to current parent/child relationships, which directly affect the psychological well being of the child and are related to college adjustment in both social and academic contexts. They found a positive relationship between authoritative parenting styles (the hallmarks being reciprocal communication and discussion with parents) and children's adjustment to college.

Hickman, Bartholomae, and Mckenry's (2000) study added further support. They found authoritative parenting and self-esteem to be significant predictors of academic adjustment of college freshmen. Allen, Moore, Kuperminc, and Bell's (1998) study identified a direct link between attachment experiences and adolescent psychosocial functioning. Rice, Cole, and Lapsley's (1995) findings linked low attachment security to multiple adjustment difficulties in college in both cross-sectional and longitudinal analyses. An interesting finding in the study was that 1/3 of students who were described as insecurely attached as freshmen were considered securely attached during junior year; this gives hope for "corrective emotional relationships". In addition, results from the study showed that students with secure attachment style manifested greater academic, emotional, and social adjustment than insecurely attached students (Vivona, 2000). Scores by gender reflected similarities.

Emotional Intelligence is a second factor that has been researched extensively. Mayer and Salovey (1997) define Emotional Intelligence as "the ability to perceive emotions, to access and generate emotions so as to assist thought, to understand emotions and emotional knowledge, and to reflectively regulate emotions so as to promote emotional and intellectual growth" (p. 22). This definition reflects the concept that intelligence and emotion are interconnected. In addition, Mayer and Salovey discuss the relationship between quality of parental attachment and emotional style. They purport that a securely attached individual experiences more freedom to explore a plethora of emotions because he or she has a strong secure base to rely upon. In contrast, the child with an anxious attachment to the primary caregiver experiences rejection instead of comfort to ease distress. This child

Intelligence is comprised of various components: emotion perception (which is the ability to identify one's own and others emotions which are garnered from situational and expressive cues), managing other's emotions (actively shielding one's own emotions in order to avoid hurting the feelings of others or bolstering the feelings of a depressed friend), and managing one's own emotions, particularly in relation to coping with negative emotional states and the strategies employed by the individual to combat them (Davies, Stankov, & Roberts, 1998; Izard, Schultz, & Ackerman; Ciarrochi, Chan, & Bajgar, 2000; Salovey, & Sluyter, 1997). Ciarrochi et al. (2000) did find gender differences in their study. Females self-reported higher levels of emotional intelligence than males. Their study supported their hypothesis that people high in emotional intelligence were more adept at establishing and keeping personal relationships and social support than those whose scores were not as high. Salovey, Bedell, Detweiler, and Mayer (2000) emphasized that individuals with emotional competencies are better able to deal with test-taking stress thereby enhancing SAT scores.

Another area of research that is believed to influence college adjustment is resiliency and personal coping mechanisms. Tross, Harper, Osher, and Kneidinger (2000) define resiliency as "the tendency to demonstrate commitment to a course of action when challenged, remain calm and emotionally stable when faced with unexpected circumstances, and rebound when faced with adversity" (p. 324). They reported that resiliency is a valid predictor of job performance in many occupations. They also observed a relationship between resiliency and organizational commitment. Roosa, Wolchik, and Sandler (1997) describe coping behavior as a characteristic which is significantly related to adjustment. Leong and Bonz (1997) studied various coping styles and their effects on college adjustment. The results showed that academic adjustment and personal/emotional adjustment were directly linked to coping strategies. A proactive coping style, which stressed a positive problem solving approach, was found to be related to academic adjustment in college freshmen. Gender differences were not found (scores were almost identical) in the domain of personal/emotional college adjustment; however, there was a slight (not significant) difference in

Rice and Fitzgerald (1995) reported that a common occurrence associated with individuals who demonstrate resiliency is their ability to establish and maintain a bond with a significant person (e.g., professor, relative, friend, etc.), and that this relationship serves as a buffer from life's adverse effects. Galambos and Leadbeater (2000) explain that an individual's ability to acquire and express resilient coping skills in the presence of adversity will depend on numerous factors such as stage of development, family relationships, and life circumstances. The concept of attachment history may underlie development of emotional intelligence and coping behavior, since early attachment has been found to influence emotional behavior. Miller (1997) reported that self-understanding was a primary attribute for effective functioning as were relationships with others. Self-understanding includes being aware of one's own strengths and weaknesses, and knowing how to accommodate and compensate for them. In addition, self-understanding is an integral factor in gaining control over diverse situations. Bray, Braxton, and Sullivan (1999) point out that stress is a byproduct of the college adjustment experience, and that one's coping mechanisms with issues such as separation from the home environment, intimate relationships, and disagreements with roommates, etc., causes some students to leave their institutions permanently. Shiner (1999) reported that resilient children have been found to score higher on intelligence, academic achievement, and social reasoning. It is for these reasons that the present study will assess stress-coping strategies.

An additional characteristic that appears to influence college adjustment is conscientiousness. Kelly (2001) explains that people high in conscientiousness are "purposeful, strong-willed, determined, careful, future-oriented, and motivated to accomplish goals" (p.15). Furthermore, conscientiousness and information acquisition have been shown to share common factors. Kelly's (2001) study hypothesized that individuals high in conscientiousness would be more proficient in knowledge acquisition on academic tasks than those who had low scores in conscientiousness, and that attention would significantly interact with conscientiousness in regards to information acquisition. Significant results were found for the former

years, found gender differences; females scored higher on academic conscientiousness than males did. Although in earlier years academic conscientiousness was only modestly related to academic achievement, academic conscientiousness was predictive of social competence. However, in later years academic conscientiousness was predictive of academic achievement. The results showed a significant correlation between childhood conscientiousness and high school GPA at graduation. The author suggested that academic conscientiousness may be more relevant to academic achievement in higher grades where teachers' expectations of strong academic students are greater.

Wolfe and Johnson (1995) report that conscientiousness has a significant positive correlation with performance in school, and that in their study, conscientiousness accounted for a greater portion of GPA variance than did SAT scores. Tross, Harper, Osher, and Kneidinger's, 2000 study found similar results. Thus conscientiousness merits further scrutiny in the college adjustment process.

Another important variable to consider when exploring the factors that impact on college adjustment, is motivation. Strange and Brandt (1999) report that the body of research conducted on motivation in the past twenty years has identified "mastery-oriented" students. These students are able to take advantage of intellectually stimulating and demanding tasks. They actively aspire to succeed in accomplishing or completing challenging assignments in which they have the opportunity to acquire new knowledge in order to elevate their competence. Their confidence and persistence are evidenced throughout their academic endeavors, and their stress level is manageable and does not hamper their successes. Davidson, Beck, and Silver (1999) report that academically successful students and students who are satisfied with college tend to be intrinsically motivated. In contrast, poor academic performance is associated with extrinsic motivation.

Cote and Levine's (2000) study found that the type of motivation a student possesses, as well as type of environment, were better predictors of academic achievement in college than were high school grades. In addition, they found that motivation was more important than intelligence in regard to

academic motivation scores than those who earned their degrees in eight semesters or more. Finally, their research showed that correlations between the Academic Motivation Scale and the academic adjustment measure were positively and statistically significant, whereas correlations between SAT scores and academic adjustment were mostly non-significant, and all correlations were lower than the Academic Motivation/academic adjustment correlates. Colquitt, LePine, and Noe's (2000) study supported the theory that motivation to learn has a greater impact on learning outcomes than cognitive ability does.

Lastly, depression is the final variable explored in this study. Martinee-Pons (1998) reported that depression symptomatology was defined as "the cognitive, affective, and somatic manifestations of dejection or despondency"(p. 8). Wintre and Yaffe (2000) report that the transition to university life can be an overwhelming experience, which precipitates emotional maladjustment and depression. Their study found that depressive symptomatology significantly predicted poorer overall adjustment for both males and females. Allen, Moore, Kuperminc, and Bell (1998) report that depression may result from attachment insecurity, which is similar to feelings of low self-worth. Bressler (2002) explored the correlations between depression, anxiety, and emotional intelligence. She reported that emotional intelligence works as a buffer or protective factor against depression because individuals high in emotional intelligence are able to perceive, appraise, and express emotions effectively. Conversely, individuals with low emotional intelligence are more prone to maladaptive emotional states because they are less likely to recognize irrational thoughts which can lead to depressive symptomatology.

Emotional intelligence is the next variable believed to affect college adjustment. Specifically, it is hypothesized that Emotional Intelligence scores will positively correlate with overall college adjustment scores. It is also hypothesized that a gender difference will be detected, that is that females will receive higher emotional intelligence scores than males will.

Resiliency has been shown to impact the college adjustment process.

adjustment scores.

Motivation is another influential variable under scrutiny. Therefore, it is hypothesized that students' motivation scores will positively correlate with overall college adjustment scores.

Lastly, the depression variable is explored. It is hypothesized that students' depression scores will inversely correlate with overall college adjustment scores. That is that the higher the depression score is the lower the overall college adjustment score will be. It is hypothesized that a gender difference will be revealed. That is that females will receive higher scores in the depression domain than males will.

Method

Participants

141 Monmouth University freshmen (22.7% males, 77.3% females) volunteered to participate in the research. 79.4% were on campus residents, 6.4% were off campus residents, and 14.2 % lived at home. Ethnic background was diverse – European Americans (74.5%), African Americans (5.7%), Asian Americans (1.4%), other (14.2%). Most students were regular admit (85.8%). There were also honors (5%), and developmental (9.2%) participants. Volunteers were entered into a raffle to win two gift certificates (\$50, \$25) to the University bookstore, and class credit was given to those students enrolled in Introduction to Psychology. All volunteers were treated in accordance with “Ethical Principles of Psychologists and Code of Conduct” (American Psychological Association, 1992).

Materials

The Attachments and Clinical Issues Questionnaire (ACIQ) by Lindberg and Thomas (2002) was intended to be used to determine the working models of attachment. For the purpose of the present study they were meant to be dummy coded as either securely or not securely attached. The ACIQ is made up of 29 scales that measure diverse measuring styles (for example, secure, avoidant, ambivalent, codependent, etc.). There are a total of 258 questions and respondents are given four choices (never, sometimes,

Scale (GEIS) by Albert Mehrabian (2001) had been chosen. This scale is composed of two components, Emotional Intelligence and Emotional Thinking. Mehrabian (2000) found that the Emotional Thinking scale (which accounts for approximately 20% of the GEIS) was a significant negative predictor of success in life. The GEIS is made up of 45 items, and participants used a 9-point agreement-disagreement scale to respond to the questions. Two students omitted one question each and a score of 0 (neutral) was inserted by the researcher. The alpha internal consistency for the Emotional Intelligence scale (first 37 items) is .85. The alpha internal consistency for the Emotional Thinking scale (remaining 8 items) is .79. The test-retest reliability for the GEIS is .72 (Mehrabian, 2001).

Resiliency and coping was measured using a two-item questionnaire designed by Stapley and Hatchard (under review). The two questions included were: What do you Usually *do* when you feel sad? , and What do you usually *do* when you feel angry? The inter-rater reliability is reported at 95% perfect agreement. Responses that indicate problem-solving or self-soothing are coded as resilient. Others, for example aggressive, destructive responses are coded as non-resilient.

The Academic Motivation Scale by Baker and Siryk (1978 version) was used to assess motivation. This 35-item questionnaire has a 19-point scale (ranging from “applies very closely to me” to “doesn’t apply to me at all”) (Baker & Siryk, 1984). One participant left out one answer and a score of 10 (precisely in the middle) was inserted by the researcher. The internal consistency reliability (Chronbach’s alpha) was reported at .88. In addition, correlations between the Academic Motivation Scale and validity criteria (academic performance, effort towards academic goals, and academic adjustment) were significantly higher than the correlations between SAT and validity criteria.

Conscientiousness was measured using the conscientiousness scale of the revised NEO Personality inventory (NEO-PI-R) (Costa, & McCrae, 1991). This scale is made up of 48 questions which measure six facets (competence, order, dutifulness, achievement striving, self-discipline) of conscientiousness. One student left out one question and a score of neutral

constitute depression. Four students failed to complete the BDI and their data on the BDI was not calculated into overall BDI scores. Two students skipped one question each, and one student skipped two questions. An average score obtained by calculating their other responses on the BDI was inserted by the researcher. The Chronbach alpha reliability for the BDI-II is approximately .93. The BDI-II has been used in numerous studies pertaining to college adjustment (Wintre & Yaffe, 2000; Davies, Stankov, & Roberts, 1998; Dawda & Hart, 2000; Vivona, 2000).

College adjustment was measured (in the Spring of 2003) using the Student Adaptation to College Questionnaire (SACQ) (Baker & Siryk, 1989) and GPA scores obtained in Spring 2003. Using GPA to assess college adjustment has become increasingly more popular as evidenced by Shiner (1999), Wolfe and Johnson (1995), and Davidson, Beck, and Silver (1999). The SACQ is a 67-item, self-report questionnaire that can be completed in approximately twenty minutes. The SACQ is comprised of four subscales; Academic Adjustment, Social Adjustment, Personal-Emotional Adjustment, Goal Commitment/Institutional Attachment. Students respond on a 9-point scale, which ranges from “applies very closely to me” to “doesn’t apply to me at all”. The alpha coefficients for reliability were as follows: Academic Adjustment .81 to .90, Social Adjustment .83 to .91, Personal-Emotional Adjustment .77 to .86, Institutional Attachment .85 to .91, and for the full scale .92 to .95. The SACQ has been widely used (Huff, 2001; Vivona, 2000; Hickman, Bartholomae, & McKenry, 2000; Rice, Cole, & Lapsley, 1990; Rice & FitzGerald, 1995; Leong & Bonz, 1997; Pancer & Hunsberger, 2000; Hertel, 2002; Baker & Schultz, 1992; Protinsky & Gilkey, 1996; Wintre & Yaffe, 2000; Gerdes & Mallinckrodt, 1994; Martin Swartz, & Madson, 1999).

Procedure

Participants were recruited from fliers distributed in the freshmen seminar classes at Monmouth University, as well as from fliers sent via e-mail to all incoming freshmen students. In addition, participants were recruited from the psychology subject pool. Students were directed to attend

informed that several questionnaires regarding emotions and college adjustment would be forthcoming. A consent form was read to the students, and after ascertaining that the students understood the form, they were asked to sign the form. In addition, participants were notified of their right to terminate the study at any time without experiencing adverse consequences. During the first two to three weeks of the semester, participants were given the GEIS, then the resiliency questionnaire, the NEO-PI-R conscientiousness scale, the Academic Motivation Scale, and the BDI. The SACQ was completed during the tenth or eleventh week of the term. No interpretations or added instructions were given to the participants. During the following semester (Spring 03) student's GPA will be obtained and used as a criterion marker for college adjustment in a longitudinal follow-up.

Results

Data Collection One: Intercorrelations Among Predictor Variables. Data Collected Between Second and Fourth weeks of Fall Semester.

Results from data collection 1 showed that those who reported resilient coping with experiences of anger were higher on GEIS (resilient $M=39.58$, $SD=29.16$; non-resilient $M=14.51$, $SD=29.77$) $t(136)=-4.80$, $p<.001$, r effect = .38.

Those who reported resilient coping with sadness were higher on motivation, (resilient $M=392.94$, $SD=64.92$; non-resilient $M=361.63$, $SD=46.67$) $t(135)=-2.02$, $p<.05$, r effect = .17.

Those who reported resilient coping with sadness were higher on emotional intelligence (resilient $M=34.42$, $SD=30.04$; non-resilient $M=8.37$, $SD=33.56$) $t(135)=-3.45$, $p<.001$, r effect = .28.

Those who reported resilient coping with sadness were higher on conscientiousness (resilient $M=118.85$, $SD=18$; non-resilient $M=107.42$, $SD=22.64$) $t(134)=-2.47$, $p<.05$, r effect = .21.

Significant gender effects were not found for depression (males $M=8.39$, $SD=6.03$; females $M=10.46$, $SD=7.68$) $t(135)=-1.38$, $p=.17$, r effect = .19.

Table 1

Simple Correlations Between Predictive Variables

| | <u>BDI</u> | <u>GEIS</u> | <u>AMS</u> | <u>NEO-PI-R</u> |
|------------------------|------------|-------------|------------|-----------------|
| Depression | 1 | | | |
| Emotional Intelligence | -.38 | 1 | | |
| Motivation | -.24 | .40 | 1 | |
| Conscientiousness | -.13 | .46 | .59 | 1 |

Data Collection Two: Analyses With Student Adjustment To College Questionnaire As Criterion. Data Collected at the End of First Semester Freshman Year.

Simple correlations were computed among all of the predictors and the scales of college adjustment, for those students who returned to complete this outcome measure (N=57). See Table 2. Based upon the correlations, standard multiple regressions were performed between SACQ total score as the criterion and conscientiousness, motivation, GEIS, BDI, and anger as the predictor variables. These predictors were included in the regression because their simple correlations with criterion were at .3 or above. Only BDI ($sr_1^2=.08$) contributed significantly to the prediction of full scale SACQ. See Table 3.

Table 3

Summary of Standard Multiple Regression Analysis for
Variables Predicting SACQ Scale (N=61)

| Variable | B | SE B | ? |
|-------------------|-------|------|-------|
| Conscientiousness | .54 | .46 | .17 |
| Motivation | .12 | .14 | .12 |
| GEIS | .26 | .27 | .15 |
| BDI | -3.11 | 1.22 | -.32* |

regression
 $p < .001$) with gender, BDI, GEIS, motivation, and conscientiousness as predictors. Gender ($sr_i^2 = .05$), BDI ($sr_i^2 = .03$), motivation ($sr_i^2 = .05$), and conscientiousness ($sr_i^2 = .04$) contributed significantly to academic adjustment. See Table 4.

Table 4

Summary of Standard Multiple Regression Analysis for Variables Predicting Academic Adjustment (N=60)

| Variable | B | SE B | ? |
|-------------------|--------|------|------|
| Conscientiousness | .33 | .15 | .27* |
| Motivation | .11 | .05 | .29* |
| GEIS | .07 | .09 | .11 |
| BDI | -.77 | .40 | -.20 |
| Angry | 6.2 | 5.5 | .12 |
| Gender | -13.40 | 5.35 | .23* |

Note $R^2 = .58$ ($p < .001$)

* $p < .05$

The overall model is significant for personal/emotional adjustment (R for regression $p < .001$) with BDI, GEIS, and conscientiousness as predictors. BDI ($sr_i^2 = .10$) contributed significantly to personal/emotional adjustment. See Table 5.

Table 5

Summary of Standard Multiple Regression Analysis for Variables Predicting Personal Emotional Adjustment (N=60)

| Variable | B | SE B | ? |
|-------------------|-----|------|-----|
| Conscientiousness | .22 | .14 | .20 |
| GEIS | .11 | .09 | .17 |

established in the current study. Thus those that were .25 or above were included in multiple regression for each of these criterion. However, significance was not found for the overall social adjustment scale model or for the overall commitment scale model, nor was significance found for any of the individual predictors.

Data Collection Three: Analyses With Spring Semester GPA As Criterion and Predictors Collected Between Second and Fourth Weeks of Fall Semester.

Simple correlations were computed among all of the predictors and Spring GPA. See Table Six. Based upon the correlations, standard multiple regression analysis was performed for Spring GPA scores.

The overall model is significant for Spring semester GPA (R for regression is $p < .001$) with BDI, conscientiousness, and motivation because their simple correlations with criterion were at .2 or above. BDI ($sr_i^2 = .03$) and conscientiousness ($sr_i^2 = .05$) contributed significantly to Spring GPA. See Table Seven.

Table 7

Summary of Standard Multiple Regression Analysis for Variables Predicting Spring Semester GPA (N=134)

| Variable | B | SE B | ? |
|-------------------|------|------|-------|
| Conscientiousness | .02 | .005 | .28 |
| Motivation | .0 | .002 | .03 |
| BDI | -.02 | .01 | -.17* |

Note $R^2 = .13$ ($p < .001$)

* $p \leq .05$

Discussion

As expected, depression was negatively correlated with motivation, and emotional intelligence. However, a significant inverse correlation was not found for conscientiousness and depression. Yet conscientiousness

AMS, CS, and GEIS, since sadness is believed to affect a broader spectrum of psychosocial emotions (Wong & Wiest, 1999).

One shortcoming in the study was the rather skewed ratio of males to females (32 males compared to 109 females). Perhaps the generalizability of the gender differences (or lack of differences) is limited in view of the uneven ratio. Further studies are deemed necessary to explore the gender issues.

The overall findings suggest that of all the variables under study, BDI appears to be the most predictive of college adjustment. This has important implications as the BDI is a brief questionnaire, easy to administer, and takes little time to complete (most students were able to complete the BDI in five to ten minutes). Since the BDI correlated with college adjustment, it is recommended as a screening tool. The potential to predict college adjustment at early opportunities should be explored, as the benefits of early intervention should prove positive for both students and the school.

Future research should include an over-sample of males, as the lack of a large number of male participants hinders the research process. Additionally, research mechanisms designed to encourage students' continued participation in the study needs to be addressed. The substantially high mortality in the sample between time one data collection of all of the predictor variables and time two data collection resulted in diminished statistical power for analyses concerning the Students' Adjustment To College Questionnaire.

It is not known whether the relationship between the predictors and the scales of Social Adjustment and Commitment to College did not reach significance due to the content of the scales or due to the small number of participants (62). It is also possible that this particular sample is not predictive of the larger population. Therefore, additional studies with larger and diverse populations could prove worthwhile.

The 29 scales and 258 questions of the Attachment and Clinical Issues Questionnaire proved to be too time-consuming to administer to students. An alternate less cumbersome testing device with comparable validity would be ideal, as parental attachment could have a substantial

from the perspective of an internal working model. Additional measurement designed to address this issue would prove beneficial.

As predicted, conscientiousness was found to be significantly related to Spring GPA. Further testing to replicate this finding would enhance the generalizability of these results. Additionally, Emotional Intelligence was not found to be significantly related to Spring GPA. Although the sample was large (141 students), perhaps the under-representation of males resulted in a lack of variability of scores, thereby obscuring effects that otherwise might have been found. Further studies with a larger male population would be informative. Resiliency was not found to relate significantly to Spring GPA. Additional research with alternate testing instruments should be explored in order to determine if indeed college adjustment is unrelated to individual coping mechanisms.

This study is an important contribution to the literature since it employs a prospective design. This allows the researchers to know that students were, for example, depressed before they performed badly academically rather than because of their performance. It is suggested that this type of design be employed further to help assess the importance of different antecedent psychosocial characteristics and states for predicting adjustment to college.

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WINDOWS OPEN AND DOORS SHUT: THE BALANCE BETWEEN COMMUNITY AND PRIVACY OF SOUL IN VIRGINIA WOOLF'S TO THE LIGHTHOUSE

Karen Mintz

Abstract

Virginia Woolf's success and universality as a writer is due in large part to her desire to recreate life as it truly exists through characters who exemplify the complex and paradoxical nature of the human condition. This trend is especially apparent in her use of window imagery in To the Lighthouse. The recurring symbol of the window serves as an illustration of the semi-permeable barrier between the individual and the community. Through the characters' emotional agonies and internal struggles, Woolf is able to emphasize the idea that the human being is so complex that one must settle for merely scratching the surface of another person, although there is always the pressing desire to achieve that moment of clarity, of complete understanding that breaks down all barriers and unites two people in the bond of awareness and knowledge. Similarly, the emphasis placed on keeping windows open and doors shut in Woolf's novels illustrates how a balance between privacy of soul and the social self is necessary to maintain relationships between human beings. By incorporating window and door imagery along with techniques such as stream-of-consciousness, Virginia Woolf is able to more fully represent the human condition and the depth and complexity of the individual.

ordinary struggles to understand the characters endure. Virginia Woolf's gift as a novelist is exemplified through her ability to understand that fiction must mirror all aspects of true humanity, no matter how imperfect or ambiguous. Woolf's desire to accurately portray life and humanity through her characters resulted in her heightened awareness of the dichotomy which is present, though in varying levels of intensity, in each and every person. Her characterization techniques and her depiction of the inner musings of her characters as they interact with one another demonstrate the striking distinction between the social person and the private person. As a member of society, each of us is aware of this distinction and ambiguity of character, and we often strive to tap into that mysterious, private realm of our peers. For example, on page 67 of To the Lighthouse, Mr. Ramsay must cope with his conflicting desire to speak to his wife and his wishes to respect her obvious need for privacy in that particular moment, "Then, he wanted to tell her that when he was walking on the terrace just now – here he became uncomfortable, as if he were breaking into that solitude, that aloofness, that remoteness of hers..." In To the Lighthouse, Virginia Woolf addresses the problem of never fully understanding one's fellow human being, and in a sense, of never fully understanding oneself. However, this problem is not seen as an issue that should ever be completely resolved because it is this constant search for knowledge and understanding that maintains the relationship between each person. Through the interactions and revelations of her characters, especially those of Mrs. Ramsay and Lily Briscoe, Woolf demonstrates that although humans may desire to gain definitive knowledge about one another, communion of different souls can be achieved more fully when the balance between society and privacy of soul is both maintained and respected. The recurring image of the window provides a means for the characters and the reader to gain knowledge and achieve a sense of understanding about themselves, their peers, and the world around them – of course this knowledge and understanding are both limited and vague, but the momentary glimpses into the minds of others are enough to keep the characters connected in a spiritual union and to keep them working toward more definitive answers about each other, themselves, and the outside world.

beauty of the world.

Virginia Woolf's desire to provide her reader with a realistic representation of life as it truly exists is a key factor in the development of many of her novels and works, such as Jacob's Room (1922), "Mr. Bennett and Mrs. Brown" (1925), and Mrs. Dalloway (1925). In 1925, two years before the publication of To the Lighthouse, The Common Reader was published. In her essay "Outlines" that is included in this collection she states,

But novelists differ from the rest of the world because they do not cease to be interested in character when they have learnt enough about it for practical purposes. They go a step further, they feel that there is something permanently interesting in character itself (195).

It is this fascination with and awareness of character that allows Virginia Woolf to create characters that are heroic in overcoming the daily agons that each of us must resolve and ones that are triumphant in simply *living* – universal accomplishments that do not require any hyperbolic grandeur in order to be duly appreciated. The use of agons in her work allows characters to engage in emotional conflicts and struggles. As they work these emotions out through reasoning, interactions with others, or their stream-of-consciousness, the characters are able to develop more fully and deeply; thus allowing the reader to achieve insight that would have been otherwise unattainable. These agons not only lead to a resolution and a deeper sense of understanding, but they also result in significant creative activity. This effect can be noted more specifically in the development and final success of Lily Briscoe's painting.

Unfortunately, not all of Woolf's readers have been able to recognize her success in undertaking the enormous feat of recreating humanity and life through a work of fiction. Virginia Woolf struck out against this misinterpretation of her technique through a variety of essays such as "Modern Fiction" (1919), in which she denounces the superfluous detail provided in the novels of Mr. Bennett, Mr. Wells, and Mr. Galsworthy

life nothing else is worth while” (148-9). She realizes that life does not happen in neatly constructed, rationally clear episodes during which stereotyped characters play out obvious lessons. Woolf explains that “life is a luminous halo, a semi-transparent envelope surrounding us from the beginning of consciousness to the end” (150). This description suggests that human beings are immersed in the world of social interactions, private reflections, day-to-day drudgery, and physical sensations simultaneously – one does not allot a certain time of day for personal reflections and a different time for interactions with the external world. As Ann Banfield remarks, “Separate moments are not real, they are imposed by the mind” (480). It is the constant interplay between the various aspects of life that maintains this busy “halo” by which we are each surrounded. Michelle Pridmore-Brown describes Woolf’s impressions of the world ““as a pulsating field of mind and matter in which everything is interconnected”” (Westling 856). Instead of being intimidated by this complex, permanent state of activity and movement, Virginia Woolf embraces it and accepts it as a tool in recreating life through her novels. She states in “Modern Fiction,”

The mind receives a myriad impressions – trivial, fantastic, evanescent, or engraved with the sharpness of steel. From all sides they come, an incessant shower of innumerable atoms...Let us record the atoms as they fall upon the mind in the order in which they fall, let us trace the pattern, however disconnected and incoherent in appearance, which each sight or incident scores upon the consciousness. Let us not take it for granted that life exists more fully in what is commonly thought big than in what is commonly thought small (150).

In order to fulfill this statement and thereby create a more accurate description of life and humanity, Woolf creates characters with depth and ambiguity that embody numerous contradictions and indecisions. In review of Jacob’s Room, her older contemporary, Arnold Bennett, stated that her characters “do not vitally survive in the mind because the author has been obsessed by details of originality and cleverness” (Bennett). Woolf was hurt

can work through his/her emotions in order to draw conclusions about a character. As the narrator imagines a history for Mrs. Brown, the reader is able to grasp the contradicting ideas and emotions that must be sorted through,

The important thing was to realize her character, to steep oneself in her atmosphere. I had no time to explain why I felt it somewhat tragic, heroic, yet with a dash of the flighty and fantastic, before the train stopped, and I watched her disappear, carrying her bag, into the vast blazing station. She looked very small, very tenacious; at once very frail and very heroic. And I have never seen her again, and I shall never know what became of her (198-9).

These varying impressions are all worked out in the narrator's emotional agon, and the reader is able to reach a greater sense of the characters' depth and thought processes. This technique is used to create more realistic characters that enable Woolf to more accurately depict the human condition. The contradictory nature of human emotions and thoughts are emphasized in many of her works. For example, on page 186 of To the Lighthouse, Woolf includes the line, "For nothing was simply one thing" in James's internal monologue. This line demonstrates her understanding and acceptance of complexity in human nature. As a result, she creates characters that are neither fully one thing nor fully another. For example,

Mrs. Ramsay supports this idea; what she feels like in and of herself is something dark, made up of intuition and free-ranging meditation. The more hard, definite, verbalizable parts of her are the parts she associates with being at the disposal of others, not with the core of herself (Nussbaum 739).

A distinct contrast is noted between the characters in social contexts and the characters involved in private reflection. It is this duality of human nature that results in speculation about and consideration of the character of one's peers.

She liked him for being so angry with Paul and Minta. She liked his awkwardness. There was a lot in that young man after all.” Lily Briscoe summarizes this indecision and inability to fully know one’s fellow man when she reflects, “She would never know him. He would never know her. Human relations were like that” (92). Lily Briscoe’s internal monologue reveals that human interactions exist primarily on the surface, with little provision for insight.

While working on Mrs. Dalloway, Virginia Woolf describes her technique in creating believable, realistic characters in the following excerpt from a diary entry, “...how I dig out beautiful caves behind my characters. I think that gives exactly what I want; humanity, humour, depth. The idea is that the caves shall connect and each comes to daylight at the present moment” (Woolf, L., Ed.). Woolf brings this idea of cavernous depth into her novels by creating characters that move and change and cannot be fully understood. This ambiguity leads to intense emotion and internal agon as one’s peers attempt to reach some sort of understanding and union of the soul with each person. On page 62 of To the Lighthouse, Mrs. Ramsay thinks to herself, “Beneath it is all dark, it is all spreading, it is unfathomably deep; but now and again we rise to the surface and that is what you see us by.” Through Mrs. Ramsay’s emotional agon and contradictory internal thoughts, Woolf is able to reiterate the idea that the human being is so complex, so obscure and out of reach, that one must settle for merely scratching the surface of another person, although there is always the pressing desire to achieve that moment of clarity, of complete understanding that breaks down all barriers and unites two people in the bond of awareness and knowledge.

Martha Craven Nussbaum comments on the evasiveness of human character, even as one considers himself,

Human beings cannot even take down the direction of their own thought, so rapidly and complexly does it move. How much more difficult, then, is it to communicate this thought to another; how impossible would it seem, by following the signs given by another to attain access to the rapid complex inner world that exists inside another body (734).

After all, “emotions don’t stand still to be inspected like so many stones or bricks. The act of bringing them to consciousness frequently changes them; the act of expressing them to another almost always does so” (Nussbaum 738). It is due in part to this insufficiency of human expression that one must maintain privacy of soul. In one’s own private emotions and thoughts, a resolution can take place through internal agon. These emotions are forced to the surface to be wrestled with and considered until a conclusion can be made. While each person takes part in these types of internal struggles in his own private thoughts, it is the mystery and speculation surrounding one’s peers that serves as a catalyst for human interactions. On page 50, as Lily Briscoe struggles to gain an understanding of Mrs. Ramsay and achieve unity of soul (which is only reached upon the completion of the painting), the narrator poses the questions,

Was it wisdom? Was it knowledge? Was it, once more, the
deceptiveness of beauty, so that all one’s perceptions, half-way to
truth, were tangled in a golden mesh? or did she lock up within
her some secret which Lily Briscoe believed people must have for
the world to go on at all?

The final remark in this excerpt emphasizes the importance of secrecy and privacy in maintaining a functioning society. If these aspects of our character were no longer evident and privacy of soul was diminished, the very nature of man would be compromised. There would no longer be a desire to speculate and investigate in an attempt to come closer to the truth about our peers. In a society without these features, all human interaction would be minimal and would not evoke emotional or intellectual stimulation.

Martha Craven Nussbaum explains that “The goal of complete transparent access to the ‘sacred tablets’ is not just unattainable, it is morally problematic, since it asks that Mrs. Ramsay surrender her privacy and her boundedness before Lily’s curious gaze” (742). Each individual is entitled to this privacy of soul, and while others may make speculations about one another, this privacy can never and *should* never be corrupted. However, while privacy must be maintained, the need for human interactions to

him out once more and as a sign of his mourning. In Mrs. Dalloway, Clarissa repeatedly goes to the window to admire the privacy of soul illustrated by the old woman across the way,

She parted the curtains; she looked. Oh, but how surprising! – in the room opposite the old lady stared straight at her! She was going to bed...She was going to bed, in the room opposite. It was fascinating to watch her, moving about, that old lady, crossing the room, coming to the window. Could she see her? It was fascinating, with people still laughing and shouting in the drawing-room, to watch that old woman, quite quietly, going to bed...She felt somehow very like him – the young man who had killed himself. She felt glad that he had done it; thrown it away (185-6).

It is at this point in the novel that Clarissa experiences the mourning process and has her epiphany about the importance of preserving one's privacy of soul.

Similarly, in To the Lighthouse, the window is a central image in resolving emotional struggles, initiating creative activity, and bringing unity of souls to the characters. Throughout "The Window" portion of the novel, the shape of Mrs. Ramsay and James in the window is central to Lily's creative activity and a focal point for each of the characters. It is this memory of the two in the window that eventually leads to Lily's completion of the mourning process and the success of her painting ten years later (201). In addition, Mrs. Ramsay experiences intense emotion at the window. Her "mental orgasm" on page 65 is a result of the beam from the lighthouse shining upon her as she sits in the window. This recurring image is emphasized and becomes central to the development of the novel as personal relationships, emotional agons, and the creation of art are considered.

Throughout Mrs. Ramsay's internal monologues in To the Lighthouse, Virginia Woolf includes variations of the phrase, "...they must keep the windows open and the doors shut" (14, 27, 49). The significance behind the recurrence of this theme can be explained in terms of human

window is a threshold that allows a regulated amount of interaction between individuals; a threshold that allows for communication without resulting in the obstruction of privacy. This threshold can only be bridged after an intense emotional struggle during which a limited sense of understanding can be achieved. In his article, "The Open Window and the Storm-Tossed Boat: An Essay in the Iconography of Romanticism," Lorenz Eitner states, "The window is like a threshold and at the same time a barrier" (286). This description suggests that while there are opportunities for communication and communion through the symbolical "open window," there is also a limit to the amount of knowledge about a person one can obtain. This idea of balance is critical. Virginia Woolf celebrates the tendency for people to try to gain an understanding of one another, but she also accentuates the need for privacy and personal reflection. Here lies the difference between the open window and the open door. A window is semi-permeable; one must put forth effort and imagination in order to draw conclusions from a distance. A door, on the other hand, allows for the direct exchange of ideas without limitation. An open window maintains balance between the social self and the private self, while an open door would leave no opportunity to protect the revered privacy of soul. Therefore, Virginia Woolf includes the repetition of the aforementioned phrase in order to emphasize the need to maintain and respect the duality of human nature.

Just as characters cannot be perceived as just one thing or another, the symbol of the window also has multiple roles in the novel. In addition to its role as a representation of the limited understanding an individual can achieve about another, the window also acts as a catalyst for creative activity, which will ultimately lead to the successful achievement of unity. At several points during the novel, Virginia Woolf has the narrator point out that certain characters are standing directly in front of the windows, obstructing the view and diminishing the opportunity for exchange of ideas and development of creativity. At the very beginning, when Mr. Ramsay and Charles Tansley crush James's hopes of traveling to the lighthouse, both men are standing in front of the window. The first instance is noticed when Mr. Ramsay speaks, "'But,' said his father, stopping in front of the drawing-room window, 'it

novel. In her book, A Room of One's Own, Woolf comments, "...for no human being should shut out the view" (114). Therefore, the reader can conclude that the negativity surrounding Mr. Ramsay's and Mr. Tansley's lack of compassion for young James is only worsened by the fact that they are, in fact, shutting out the view.

This relationship between negativity and the obstruction of view is continued during the dinner party. Before the guests are united and enjoying themselves, Charles Tansley feels a great deal of animosity toward everyone. Woolf combines the interior monologues of Charles Tansley and Lily Briscoe to appropriately illustrate his bitterness and her negative response toward him:

What damned rot they talk, thought Charles Tansley, laying down his spoon precisely in the middle of his plate, which he had swept clean, as if, Lily thought (he sat opposite to her with his back to the window precisely in the middle of view), he were determined to make sure of his meals. Everything about him had that meager fixity, that bare unloveliness (85).

Once again, Woolf makes a pointed effort to specify that Charles Tansley is seated directly in front of the window, obstructing the view and limiting creativity and the exchange of ideas. It is this very detail that is revisited as Lily Briscoe comes closer to her artistic epiphany at the end of the novel. As she remembers Charles Tansley, Lily specifically recalls his tendency to sit "right in the middle of the view" (159) at dinner. Virginia Woolf associates the obstruction of view with negative behaviors and attitudes in order to further emphasize the importance of the open window as a means of reaching the truth and immersing oneself in creativity.

Unity is not achieved until the candles are lit and the light is reflected in from the windows, enclosing everyone from the outside world. The scene that is created by the glowing candles and the reflections from the windows allows Mrs. Ramsay to experience a moment of clarity depicted on pages 106-7. The narrator explains,

said was like the movement of a trout when, at the same time, one can see the ripple and the gravel, something to the right, something to the left, and the whole is held together; for whereas in active life she would be netting and separating one thing from another...now she said nothing. For the moment, she hung suspended.

In this scene, the windows have provided completion and communion between the characters, and by doing so, they have allowed Mrs. Ramsay to immerse herself in creativity and free thought, which results in her momentary sense of clarity about her family and guests. Her emotional agon, which led to this artistic success, resolves all of the unrest and struggle that was formerly present at the table. Therefore, the windows provide a symbol of the unity that has been achieved through this resolution process.

Finally, the imagery of the window serves as a means of illustrating the shift in time from pre-war Romanticism to the more fragmented, alienated views of Modernism, and ultimately to a collaboration of the two that results in the achievement of unity. In the first part of the novel, entitled "The Window," Virginia Woolf includes Romantic ideals and uses the symbol of the window to encompass these ideals. An emphasis is placed on the individual's relationship with the natural world. This emphasis is consistent with Virginia Woolf's own relationship to the external world, as is described in Randi Koppen's article, "Embodied Form: Art and Life in Virginia Woolf's To the Lighthouse." Koppen notes, "Consistently in 'A Sketch [of the Past],' one is made aware of an extraordinary susceptibility to external stimuli, a somatic, neurological experience of sound and smell, as in the memory of the garden at St. Ives" (379). Mrs. Ramsay also demonstrates this susceptibility to the external stimuli present in the natural world. Often, Mrs. Ramsay is portrayed looking out of the window and appreciating the beauty of the world. According to Lorenz Eitner,

The pure window view is a romantic innovation...The situation closes resembles a favorite theme in romantic literature: the poet at the window surveys a distant landscape and is troubled by a desire to escape from his narrow existence into the world spread

rejoices in the opportunity to merely be herself – “a wedge-shaped core of darkness, something invisible to others.” In addition to her desire to escape from her public self to enjoy her privacy of soul, Mrs. Ramsay also needs to escape from uncomfortable situations with her husband. For example, on page 123, she is unable to tell Mr. Ramsay that she loves him. As a result, “she stood at the window with the reddish-brown stocking in her hands, partly to turn away from him, partly because she remembered how beautiful it often is – the sea at night.” This scene demonstrates how the window is used during emotional struggles as it was during the Romantic period to provide an escape to the individual and to allow the individual to appreciate the beauty of the world. This outlet allows her the moments that she needs to work through her internal agon and to ultimately achieve a more peaceful state as she resolves her emotional unrest, which is symbolized through her reading of the Shakespearean sonnet.

In addition to her tendency to turn to the outside world as a means of escape, Mrs. Ramsay is depicted as a romantic, pre-war figure in that she has a remarkable appreciation for nature and she is driven greatly by her reactions to external stimuli. For example, the reader is able to notice her appreciation for nature in contrast to Mr. Ramsay’s preference for fact and intellect as the couple walks outside together. The narrator describes Mrs. Ramsay’s internal monologue,

And looking up, she saw above the thin trees the first pulse of the full-throbbing star, and wanted to make her husband look at it; for the sight gave her such keen pleasure. But she stopped herself. He never looked at things. If he did, all he would say would be, Poor little world, with one of his sighs (71).

Mr. Ramsay is unable to appreciate the beauty of nature with the same enthusiasm. The reader is also able to notice Mrs. Ramsay’s communion with nature at the very end of “The Window” as she is reflecting on her reconciliation with her husband, “And smiling she looked out of the window...(thinking to herself, Nothing on earth can equal this happiness)” (124). Mrs. Ramsay immerses herself in her emotion through her

during this section of the novel. Virginia Woolf understands that Mrs. Ramsay can no longer survive in a world where the beauty of the world provides no comfort. During the chapter when Mrs. Ramsay dies, Woolf includes a poignant reference to the window,

But alas, divine goodness, twitching the cord, draws the curtain; it does not please him; he covers his treasures in a drench of hail, and so breaks them, so confuses them that it seems impossible that their calm should ever return or that we should ever compose from their fragments a perfect whole or read in the littered pieces the clear words of truth (128).

As the war rages on and romantic idealism is destroyed, individuals such as Mrs. Ramsay can no longer seek answers in a comforting, compassionate natural world. In her article, "Virginia Woolf and the Flesh of the World," Louise Westling describes the passage on page 134 in her statement, "This flat denial signals the end of Romantic optimism and ushers in another succession of seasons and the return of spring, this time strange, eyeless, and terrible" (863). This description of the lack of compassion in nature and the harshness of spring echoes T.S. Eliot's The Wasteland as both illustrate disillusionment and reject the ideal of communing with nature to arrive at truth and peace. However, Virginia Woolf does not strip her novel of all optimism and hope in nature. Once again, she uses the symbolic window to restore some semblance of optimism. On page 141, Woolf creates this scene:

...which the ear strains to bring together and is always on the verge of harmonizing, but they are never quite heard, never fully harmonized, and at last, one evening, one after another the sounds die out, and the harmony falters, and silence falls. With the sunset sharpness was lost, and like mist rising, quiet rose, quiet spread, the wind settled; loosely the world shook itself down to sleep, darkly here without a light to it, save what came green suffused through leaves, or pale on the white flowers in the bed by the window.

solely a Romantic symbol or a Modern symbol. Instead, it serves as a combination of the two as the characters search for unity in a state of fragmentation and alienation. When the characters arrive at the house, peace has been restored, and the window serves as a means to reestablish a relationship between the individual and nature. On page 142, the narrator states, "Through the open window the voice of the beauty of the world came murmuring...entreat[ing] the sleepers...at least to lift the blind and look out." Similarly, on page 149, the tension between Cam and Mr. Ramsay is dissipated as she glances out the window and is able to appreciate the beauty of the world, "It was a wonderful night, starlit; the waves sounded as they went upstairs; the moon surprised them, enormous, pale, as they passed the staircase window. She slept at once." In both of these scenes, the window plays a major role in restoring peace and calm to individuals who have experienced a great deal of stress and loss.

However, in this final section of the novel, the window is no longer used merely to gaze out on nature. Instead, it is depicted as a threshold for exchange from the inside out and vice versa. Throughout the novel, Lily Briscoe undergoes numerous agon as she reaches for her creative epiphany. As she first embarks on her journey, she remarks, "I'm in love with this all" (19) as she motions to the hedge, the house, and the children. Her intense emotions must be brought forward into conscious thought and expressed through her artwork. She is forced to work through her love and passion for the Ramsays and their world in order to resolve her emotional struggles and complete her work of art. As she attempts to express her emotions, she is forced to go through several stages. During the first stage, she realizes the importance of an "august shape; the shape of a dome" (21) to represent the centrality of Mrs. Ramsay and James in her painting. As she comes closer to finishing the painting, her emotional agon becomes increasingly intense. During her call of despair, the reader is aware of her inner thoughts as she struggles for the privacy of soul required to resolve her emotional agon,

Such were some of the pieces, but how to bring them together?

She asked as if any interruption would break the frail shape she was building on the table. She turned her back to the window; but

creative energy increases, she is forced to delve into her emotions and truly *feel* the pain of mourning, the pain of mental illness, the struggles that each of the family members have had to undergo in order to truly express herself through the painting and achieve her work of art. Her struggle and pain during this call of despair are accentuated by her cry, “Mrs. Ramsay! Mrs. Ramsay!” (202) in an attempt to capture her emotions and rely on Mrs. Ramsay as a muse for her creative activity. Lily Briscoe’s epiphany finally comes at the end of the novel as a result of the movement she observes in the window where Mrs. Ramsay had once sat with James ten years earlier:

Suddenly the window at which she was looking was whitened by some light stuff behind it. At last then somebody had come into the drawing room; somebody was sitting in the chair. For Heavens sake, she prayed, let them sit still there and not come floundering out to talk to her. Mercifully, whoever it was stayed still inside; had settled by some stroke of luck so as to throw an odd-shaped triangular shadow over the step (201).

At this point in the novel, the window not only serves as a means of escape or a means to search for truth in nature. The window becomes a source of knowledge and reciprocation through the developed understanding of the individual character in relation to the external world. Through Lily’s pain and emotional struggle as she sought her artistic epiphany, resolution is achieved for all of the characters’ agons. Just as Mrs. Ramsay’s artistic success at the dinner table during “The Window” section of the novel was able to bring unity of soul and resolution to the characters at the dinner party, Lily Briscoe’s completed artistic success brings unity of soul to the characters present at the end of the novel. Just as the painting is finished, the boat finally reaches the lighthouse and resolves the struggles that were present throughout the entire novel (208-9). Unity and peace are restored, and the painting serves as a window or threshold that embraces and celebrates the emotional growth and resolution that has taken place from the beginning to the end of the journey.

Virginia Woolf’s desire to create an accurate representation of life

peers, and the surrounding natural world. Virginia Woolf is successful in integrating this symbol throughout the novel because she realizes that it cannot remain purely constant in a world of change. Just as the characters assume various roles and consider various internal musings, the window also varies with time and situations. As a result, Virginia is able to make a powerful statement about human nature's desire to constantly gain knowledge and the importance of maintaining a balance between community and privacy of soul.

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