



Qualitative decision making with integrated systems design methodology: A case study

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This paper is dedicated to my precocious son, Ocean Willow, for his love, cooperation, and patience throughout the completion of this research and case study manuscript.

Abstract

System design is associated with building a formal morphology of the system, comprising the process of defining the system parameters, submodels, and criteria, selecting an *optimal* candidate, and preparing a detail implementation plan for the chosen candidate. It is primarily a *wholistic* approach to engineering a system, and is differentiated from operations research and other analytical methods. Specifically, operations research may be used to help resolve problems within a system, but usually does not replace the design and development process necessary to complete a system. However, it assists in the development process of the resulting configurations.

This case study introduces a set of *criterion function models* and algorithms for the *design of systems*, along with *mathematical foundations* for building the models. Criteria for the system are determined and candidate system values are represented as probabilities in the [0, 1] range in order to achieve ordinal ranking of candidates on a *cardinal* scale. Both *mutually exclusive* and *interactive* design criteria could be represented by the developed models, which allow reflection of the nature of a wide variety of real-world systems. The developed models and algorithms are illustrated with a numerical example.

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1. Introduction

A *system* is defined as a combination of elements and components – mechanical, electrical, thermal, hydraulic, pneumatic, and even people and information – synthesized into a complex for performing a function and satisfying a need (Willow, 1999). Consequently, *systems engineering* (SE) is a comprehensive approach to the physical realization of equipment, human resources, and information. It is a *wholistic* approach to engineering a system, and is differentiated from operations research and other analytical methods and techniques. Specifically, operations research may be used to help resolve problems within a system, but usually does not replace the design and development process necessary to complete a system. However, it does aid in the development process of the resulting configurations. The conceptualizing, analyzing, and capacity-size synthesizing, of a scheme of elements may be called *systems design* (SD), whereas the whole of what must be done to bring it to reality – planning, design, construction, and initial operation – may be referred to as SE (Willow, 1999). In essence, the design of systems is defined as ‘a set of purposeful activities that are required to be defined and undertaken to achieve the system objectives’. However, the term SD and SE have been interchangeably used among practitioners, with the progression of recent SE research. Fig. 1 represents a *wholistic* approach to engineering a system.

A number of conventional approaches adhering to the rigors of mathematics implicitly assume that the system decision variables, primary and/or auxiliary, are available at the initial construction of the system model. However, this is seldom true in practice, and a number of quantitative models proceed successfully to qualify for the verification and validation (V&V) tests with *incorrect* set of variables. Verification and validation account for the robustness of the system or model in general, and does not test for the variables themselves with the premise that the model builder had been fully competent in terms of his/her choice of the decision variables. In Ostrofsky (1977), however, the importance of variable selection has been emphasized to a greater depth, and as an extension, a two-phase V&V is proposed in Willow (1999). That is, a V&V specifically for the decision variables *per se*, and another V&V for the developed model or systems design itself are implemented. In effect, only those variables which have been screened and examined to be accurately representing the system characteristics (*viz.* that have passed the first V&V) will be used in the design process.

When large-scale systems are approached, decisions made by the designer–modeler–planner almost always are made under the limitations of incomplete data, inadequate information, and the *uncertainties* associated with the randomness of the processes or activities under study. These limitations usually exist to some extent in each decision step of the process, and hence are compounded as many times as there are steps in the process by the degree of inaccuracy or uncertainty associated with each decision. Thus, it is relatively easy to understand why any process can produce results which, at best, minimize the risks associated with decisions made

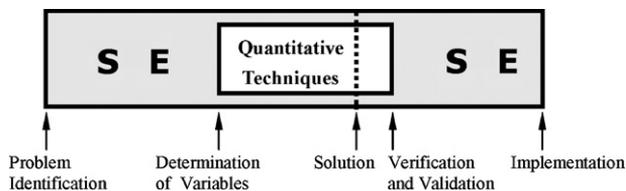


Fig. 1. Systems design engineering.

under uncertainties. Hence, there is a constant need to develop an integrated, systematic, yet robust methodology for the design of systems.

A *structured* method to designing a large-scale system had been originally proposed by Asimow (1962), and was later refined and extended by Ostrofsky (1977) and Willow (1999). As a result, a waterfall model entitled *integrated systems design methodology* (ISDM)[©] was developed. Section 2 introduces the ISDM in further detail. ISDM is distinguished from traditional and even the latest structured design techniques such as IDEF suite of methods (Mayer et al., 1995) in that it does not exclusively restrict itself as a modeling method. Instead, Willow (1999) has conceptualized a set of probabilistic *criterion models* for selecting the approximate (*viz.* optimal) alternative candidate system within ISDM, given a set of scenarios to meet the design objectives. A *design criterion* is a global attribute of the system, usually mapped from directly measurable *design parameter(s)*. Illustrations of criteria and parameters follow in Section 3. Once the classes of criterion models are identified in conjunction with their relative weights, they then constitute a trade-off, evaluation function to attain an ordinal ranking of the candidates on a cardinal scale.

One may argue that a strict cascade model such as ISDM is generally deficient in complexity and concurrency in modeling systems. However, the waterfall model provides a natural platform in expressing both information and functional flow in *designing* a system. Moreover, it is still one of the better models in representing system characteristics such as iteration (*i.e.*, structured design). Whether to select the waterfall, functional, or behavioral models such as object-oriented paradigm (OOP) (Willow, 1998a,b) depends purely on the purpose of the model and the viewpoint of the system designer. In essence, both *pros* and *cons* exist for models of all nature. OOP suits the information systems most, in which concurrency in graphical user interface (GUI) and window operations is of utmost importance.

Previously introduced design methods such as Suh's Axiomatic Theory (1990), design for manufacture (DFM) by White (1998), and Siddall's Engineering Design (1982, 1983) have the notion of application-oriented approach, and indeed were reported to be employed exclusively for certain specialized areas. In contrast, the ISDM is designed to suit all kinds of generic applications, ranging from aerospace, financial, manufacturing, logistics, and information, among others.

The objective of this case study is to describe the multiple criterion models with mutually exclusive or *interactive* criteria for generalized systems design. The eight classes of criterion function models are delineated in Section 5, preceded by theoretical insights in Section 4.

2. Integrated systems design methodology

A *system* accomplishes a phenomenon that is necessary to meet the needs efficiently. Hence, *design* (of systems) may be specified as 'purposeful planning as revealed in, or inferred from, the adaptation of means to an end, or the relation of parts to a whole' (Ostrofsky, 1977). Figs. 2–11 represent a modified *cascade model* of the systems development or project lifecycle by Willow (1999) in the designer–planner's viewpoint. The blocks with shadows represent further decomposition(s), and are depicted in the subsequent figures.

The purpose of the *feasibility study* is to produce a set of useful solutions in meeting the needs of the system (Fig. 3). It is further decomposed into needs analysis, problem identification and formulation, synthesis of solutions, and the screening of candidates (Fig. 5). *Candidate systems* or candidates in short, are the possible alternatives at hand for comparison. A single *optimal* candidate is selected, as a result of the proposed integrated system design method. Henceforth,

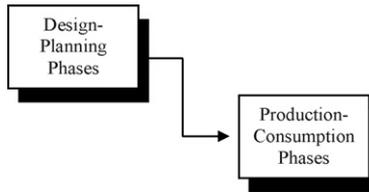


Fig. 2. Phases of the integrated systems design methodology (Level-0).

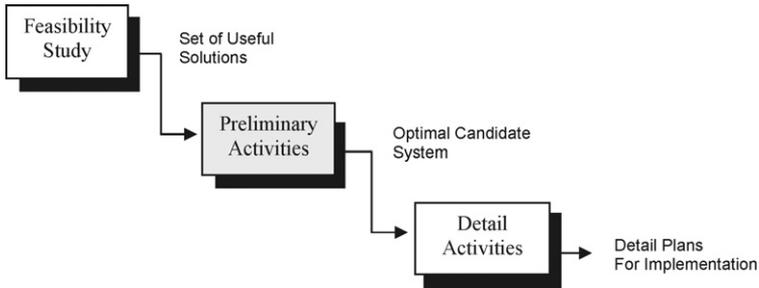


Fig. 3. Decomposed steps of the primary design-planning phases (Level-1).

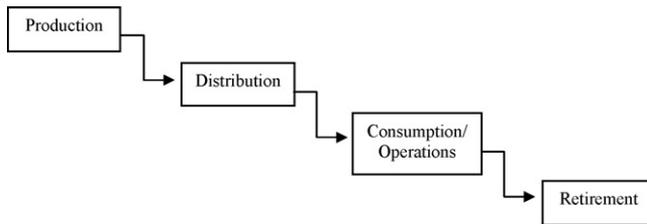


Fig. 4. Decomposed steps of the production-consumption phases (Level-1).

the adjective *optimal* and *optimum* will strictly be distinguished. Optimal systems are the local optimum which provides *approximate* solutions in a reasonable amount of time, as opposed to the global optimum, which refers to a universally proven unique, best, *exact* solution. In practice, systems scientists, systems design engineers, and simulation experts more often rely on the

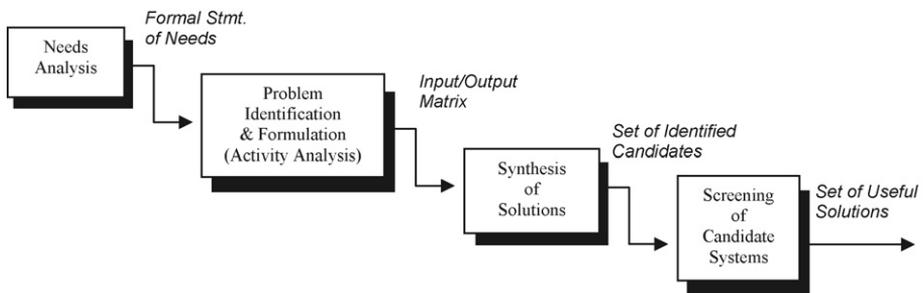


Fig. 5. Decomposed steps of the feasibility study (Level-2).

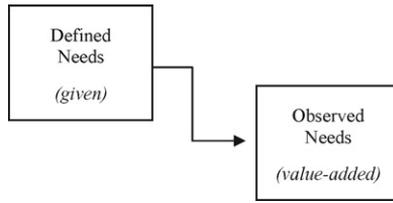


Fig. 6. Decomposed steps of the needs analysis (Level-3).

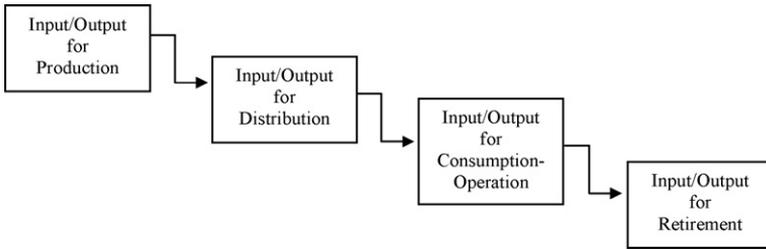


Fig. 7. Decomposed steps for the problem identification and formulation (Level-3).

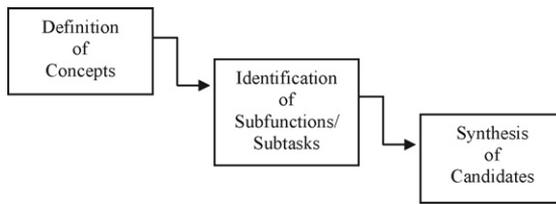


Fig. 8. Decomposed steps of the solutions synthesis (Level-3).

optimal solutions, while the majority of mathematicians and operations researchers emphasize the implications of the theoretical, *optimum* systems.

It is in the *preliminary design* activities of Fig. 3 in which the optimal candidate system is identified from the set of candidates defined in the feasibility study. Each preliminary activity is crucial to the success of systems design, since the selection of the optimal candidate is the crux of the entire design process. Subtasks of the preliminary design are depicted in Fig. 10. Note the *criterion function methods* pertain to the activities, ‘criterion modeling’ and ‘criterion function formulation’.

Once the *optimal* system is selected from the generated set of candidates, detail design for the chosen candidate follows. Fig. 11 details the associated activities.

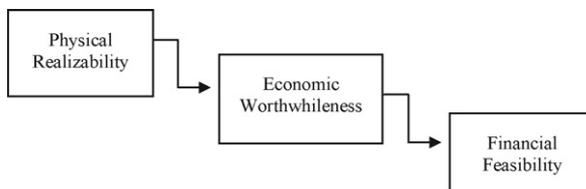


Fig. 9. Decomposed steps of the candidates screening (Level-3).

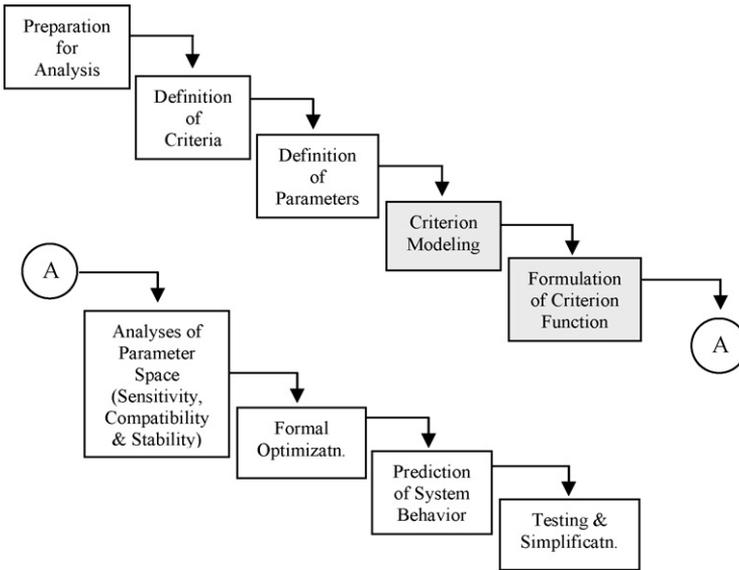


Fig. 10. Decomposed steps of the preliminary activities (Level-2).

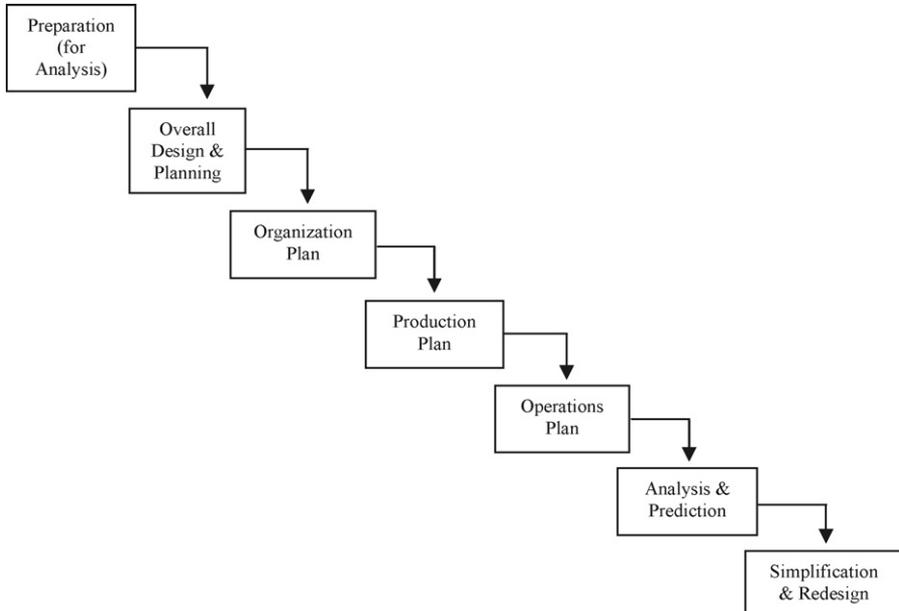


Fig. 11. Decomposed steps of the detail activities (Level-2).

3. Terminology and nomenclature

Definitions for the terms and symbols used in this paper follows (Willow, 1999).

A. Subscripts and superscripts

- i index for the criteria and relative importance/weight associated
- j subscript for submodels
- k subscript for parameters
- v superscript for *intervals* in the range of criterion i ; $v = 1, \dots, \xi_i$; applicable for Models II, IV, VI, and VIII (Fig. 14)
- V superscript for intervals of the system model; $V = 1, \dots, T$; for II, IV, VI, and VIII
- α candidate system or alternative index

B. Parameters

- m total number of candidates or alternatives for the (design–planning) model; $\alpha = 1, \dots, m$
- n total number of criteria in the model; $i = 1, 2, \dots, n$
- T total number of *intervals* of the system model; $T = 1$ for Models I, III, V, and VII; varies for Models II, IV, VI, and VIII

C. Variables

- x_i i th criterion. A (global) attribute of the system. Examples of criteria comprise economic value of a system, customer satisfaction, and so forth

$R(x_i)$ range of x_i ; $R(x_i) = x_{i,\max} - x_{i,\min}$

- \mathbf{X}_i normalized x_i in the *probability* domain. Two primary methods of normalization are:

(1) Linear interpolation (*absolute*):

$$\mathbf{X}_i = \frac{x_i - x_{i,\min}}{x_{i,\max} - x_{i,\min}} = \frac{x_i - x_{i,\min}}{R(x_i)}$$

(2) *cdf* method (*relative/statistical*):

- i classical—probability distribution fit, followed by goodness-of-fit (GOF) tests, such as the Kolmogorov–Smirnov (K-S) or Chi-square tests. *Statistical independence* among data is assumed;
- ii empirical—direct samples from data; $\Pr\{E\} = \text{Occurrence}(E)/\Omega$, where Ω is the sample space.

a_i relative importance or weight of the i th criterion

ξ_i number of *intervals* for the i th criterion; for Models II, IV, VI, and VIII (Fig. 14)

β_i^v variable for designating the v th interval for the i th criterion; applicable for Models II, IV, VI, and VIII

y_k k th parameter. A parameter is the (primitive) attribute or feature of the model. It should be *directly* measurable, and should be compact, consistent across all candidates, and complete (*viz.* exhaustive)

z_j j th submodel. A submodel is a set of mapped parameter(s)

Fig. 12 shows the relations among parameters, submodels, and criteria.

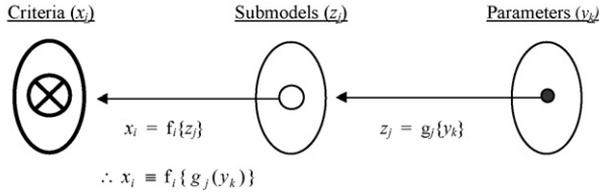


Fig. 12. Binary mapping of criteria, submodels, and parameters.

B^V variable for designating the V th interval for the model; applicable for Models II, IV, VI, and VIII

CF_α criterion function for the α th candidate or alternative;

$$\begin{aligned}
 CF_\alpha &= h_\alpha\{a_i, x_i\} = h_\alpha\{a_i, f_i\{g_j(y_k)\}\}^1 \Rightarrow h_\alpha\{a_i, \mathbf{X}_i\} \\
 &= h_\alpha\left\{a_i, \frac{f_i\{z_j\} - x_{i,\min}}{x_{i,\max} - x_{i,\min}}\right\} \\
 &= h_\alpha\left\{a_i, \frac{f_i\{g_j(y_k)\} - x_{i,\min}}{x_{i,\max} - x_{i,\min}}\right\} \\
 &= h_\alpha\left\{a_i, \frac{\prod_{\forall j} g_j(y_k) - x_{i,\min}}{x_{i,\max} - x_{i,\min}}\right\} : f_i = \Pi \\
 &= \sum_{\forall i} a_i \left[\frac{\prod_{\forall j} g_j(y_k) - x_{i,\min}}{x_{i,\max} - x_{i,\min}} \right] : h_\alpha = \Sigma
 \end{aligned}$$

4. Theoretical insights

4.1. Mathematical foundations

Mathematical models provide insights into how physical systems react to input stimuli (Hillier and Libermann, 1988). Characteristic system responses are ‘measurable parameters’ (Willow, 1999). A subset of these provides the means by which an optimal system may be selected. A mathematical model generates a set of measurements for each system. These are arguments of a decision algorithm which calculates an overall figure of merit for each candidate system or alternative. A mathematical model is an abstraction of reality, and may not precisely emulate its real counterpart (Taha, 1992). However, it does emulate the characteristics of choice. Mathematical models must be designed to an appropriate level of fidelity, since a majority of erroneously drawn conclusions are attributed to low fidelity. In addition, they often carry the inability to encompass ‘system dynamics’ required for such systems as information.

Set theory plays a crucial role in formulating the parameters and mapping them onto the subset of criteria (Willow, 1999). In turn, members of the criteria set are mapped onto the [0, 1] probability scale (i.e., normalized criteria set), in which the probability set theory (Casella and Berger, 1990) is in effect. That is, the criteria behave under the premise of the probability axioms (Hogg and Craig, 1978). These are illustrated in Fig. 13.

¹ For mutually exclusive criteria, x_i , only. Criterion interactions will involve $a_{ij} \dots$ and $x_{ij} \dots$

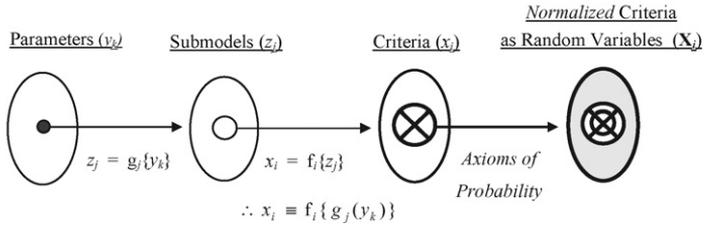


Fig. 13. Mapping the criteria into probability set.

According to Casella and Berger (1990) the ‘Axioms of Probability’ defined by Kolmogorov are:

a. $P(E) \geq 0$, where E is an event. (1)

b. $P(S) = 1$, where S is the sample space. (2)

c. If E_1, E_2, E_3, \dots are mutually exclusive or pairwise disjoint events, then

$$P(E_1 \cup E_2 \cup E_3 \cup \dots) = P\left(\bigcup_{i=1}^{\infty} E_i\right) = P(E_1) + P(E_2) + P(E_3) + \dots = \sum_{i=1}^{\infty} P(E_i) \quad (3)$$

‘Statistical independence’ for two or more events, E_i is specified as

$$P\left(\bigcap_{i=1}^n E_i\right) \equiv \prod_{i=1}^n P(E_i) \quad (4)$$

For example, when two events E_1 and E_2 are statistically independent, then

$$P(E_1 \cap E_2) \equiv P(E_1) \cdot P(E_2) \quad (5)$$

holds. Then, the probability for the set union becomes

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = P(E_1) + P(E_2) - P(E_1) \cdot P(E_2) \quad (6)$$

Hence, based on Eqs. (3) and (6), if two events E_1 and E_2 are disjoint and (statistically) independent at the same time, then either $P(E_1) = 0$ or $P(E_2) = 0$ has to hold. That is, $P(E_1 \cap E_2) = 0$, if events E_1 and E_2 are mutually exclusive. Alternatively, if both $P(E_1) > 0$ and $P(E_2) > 0$, then events E_1 and E_2 cannot be disjoint and independent, simultaneously. Verbally put, if two events, say, E_1 and E_2 are statistically independent, it implies that E_1 happening has no relation (nor any impact) whatsoever with the occurrence of E_2 and vice versa. Mutual exclusiveness or disjoint events, on the other hand, suggests that E_1 and E_2 have no common ground. Therefore, it is not uncommon to encounter random, stochastic experiments in practice, where multiple events are joint (viz. interactive) but independent. In essence, mutual exclusiveness does not suggest statistical independence.

Events were assumed to be statistically independent in this paper, but often mutually exclusive (Willow, 1999) to accommodate the characteristics of various systems in practice with real-world

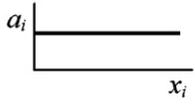
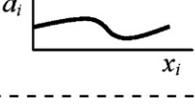
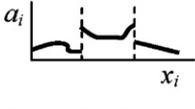
Characteristics of Criterion Interactions Nature of Relative Weights		STATISTICALLY INDEPENDENT (\mathbb{L}) & MUTUALLY EXCLUSIVE $P\left(\bigcap_{i=1}^n E_i\right) = 0$	STATISTICALLY INDEPENDENT (\mathbb{L}) & INTERACTIVE $P\left(\bigcap_{i=1}^n E_i\right) = \prod_{i=1}^n P(E_i)$	Graphical Representation of Relative Weights
DISCRETE	CONSTANT	I	V	
	INTERVAL	II	VI	
CONTINUOUS	VARIABLE	III	VII	
	VARIABLE WITH DISCONTINUITIES	IV	VIII	

Fig. 14. Classes of criterion function models.

data. That is to say, criteria for classes of Models I–IV (Willow, 1999) are mutually exclusive and those of Models V–VIII are assumed to be interactive. Fig. 14 follows to illustrate.

4.2. Multivariate statistics

Multivariate statistics is an extension to that associated with univariate, or single random variable, thus far discussed. In other words, a *random vector*, X , which is a tuple of ordered probabilities, X_i , supersedes the single variable X . A random vector is defined as

$$X = \langle X_1, X_2, X_3, \dots, X_n \rangle, \tag{7}$$

and follows the statistical properties of the univariate X . However, unlike in the univariate setting, extra care has to be taken for variable *interaction*(s). For example, a *joint* cumulative distribution function (*cdf*) for X in \mathbb{R}^n range can be represented as

$$\begin{aligned}
 F(\mathbf{X}) &\equiv F_{X_1, \dots, X_n}(x_1, \dots, x_n) \equiv P\{X_1 \leq x_1, \dots, X_n \leq x_n\} \\
 &\equiv \int_{-\infty}^{x_n} \dots \int_{-\infty}^{x_1} f_{X_1, \dots, X_n}(x_1, \dots, x_n) dx_1 \dots dx_n
 \end{aligned}
 \tag{8}$$

where $f_{X_1, \dots, X_n}(x_1, \dots, x_n) = f(x_1, \dots, x_n)$ is the joint probability density function (*pdf*).

A marginal *cdf* for a (uni-)variable X_i , on the other hand, might be acquired from (8), such that

$$\lim_{x_1 \rightarrow \infty} \dots \lim_{x_{i-1} \rightarrow \infty} \lim_{x_{i+1} \rightarrow \infty} \dots \lim_{x_n \rightarrow \infty} F(x_1, \dots, x_i, \dots, x_n) \equiv F_{X_i}(x_i), \quad -\infty < x_i < \infty \quad (9)$$

By analogous token, the marginal *pdf* for a (random) variable, X_i , could be obtained from the joint *pdf*, as described in Eq. (10).

$$\begin{aligned} f_{X_i}(x_i) &= \frac{\partial F(\infty, \infty, \dots, x_i, \dots, \infty)}{\partial x_i} \\ &= \int_{x_n=-\infty}^{\infty} \int_{x_{n-1}=-\infty}^{\infty} \dots \int_{x_{i-1}=-\infty}^{\infty} \int_{x_{i+1}=-\infty}^{\infty} \dots \int_{x_1=-\infty}^{\infty} f(x_1, \dots, x_i, \dots, x_n) \\ &\quad \times dx_1 dx_2 \dots dx_{i-1} dx_{i+1} dx_{i+2} \dots dx_n \end{aligned} \quad (10)$$

4.3. Interrelated variables

In section 4.1, the notion of *disjoint* variables (*i.e.*, mutually exclusive), *joint* variables, and *statistically independent* variables were discussed and compared. We have realized that if any number of multiple events, hence random variables, are joint (*i.e.*, interactive) and independent, then the relation in (4) holds. Since *pdf*'s and *cdf*'s are probabilities by themselves, a joint *pdf* for *independent* variables is simply a *product* of the marginal *pdf*'s, and the same holds for *cdf*'s. Namely,

$$f_{X_1, \dots, X_n}(x_1, \dots, x_n) = f(x_1, \dots, x_n) = f(x_1) \cdot f(x_2) \dots f(x_n) \quad (11)$$

and

$$F_{X_1, \dots, X_n}(x_1, \dots, x_n) = F(x_1, \dots, x_n) = F(x_1) \cdot F(x_2) \dots F(x_n) \quad (12)$$

If the variables are not independent (*i.e.*, *dependent*), they are said to be ‘correlated’. The process of obtaining the correlation of two or more variables can become extremely complex, especially with the increase in the number of variables. At times, it can be computational intractable. Three common methods practiced by statisticians in determining variable correlations are:

- (1) By using *covariance*. A ‘covariance’ of multiple variables is defined as the statistical expectation for a product of the difference between the variable itself and its mean. Eq. (13) illustrates.

$$\text{Cov}\left(\bigcap_{i=1}^n X_i\right) = \sigma\left(\bigcap_{i=1}^n X_i\right) \equiv E\left[\prod_{i=1}^n (X_i - \mu_{X_i})\right] = E\left[\prod_{i=1}^n (X_i - EX_i)\right] \quad (13)$$

- (2) By employing *correlation coefficient*. A variation of covariance with the objective of attaining a normalized, *scale-free* measure is the correlation coefficient (ρ).

$$\rho_{\bigcap_{i=1}^n X_i} = \rho_{X_1, \dots, X_n} \equiv \frac{\text{Cov}\left(\bigcap_{i=1}^n X_i\right)}{\prod_{i=1}^n \text{SD}(X_i)} = \frac{\sigma\left(\bigcap_{i=1}^n X_i\right)}{\prod_{i=1}^n \sigma(X_i)} \quad (14)$$

- (3) By adopting *linear* and *multiple regression models*. The regression models are one of the non-parametric statistical methods for data analysis (Kleinbaum et al., 1988). In particular, they are known as one of the most effective methods in determining and/or collecting correlated data. However, an insurmountable computational complexity may result, as the number of variables is increased.

Variables or criteria are assumed *not* to correlate in this paper. Correlated criteria, instead, are reserved for further research and possible future extensions to this study, such as constructing eight additional classes of models with the statistical-independence restriction relaxed.

5. Criterion function models for interactive criteria

Section 2 described the processes of the *integrated systems design methodology*, generalized by Willow (1999). In particular, the *criterion function*, CF_{α} , makes possible a quantitative formulation of the design–planning objectives. This effort, then, will be directed towards the examination of a substantiated method for combining criteria into a quantitative function which enables the designer–planner to evaluate candidate systems, so that an *ordinal ranking* is achieved which results from evaluating alternatives against a *cardinal scale*. Willow (1999) further notes that the *necessary conditions* for accomplishing this evaluation are:

- (1) A *feasibility study* has been accomplished. Specifically,
 - a. The needs analysis has been accomplished.
 - b. The design–planning problem has been identified explicitly from the activity analysis to the point where design–planning criteria have been established, and acceptable ranges for the criteria identified.
 - c. A set of useful solutions to the design problem has been synthesized.
- (2) The *preliminary design* has progressed to the stage where analytical models of the criteria have been formulated, in terms of system design parameters. The depth to which the criteria are modeled from design parameters is assumed to be adequate for the ensuing computational procedures.

The eight possible classes of criterion function models were illustrated earlier in Fig. 14. Enhanced algorithms for the models are developed in selecting the optimal candidate system. Criteria for the system are determined and candidate system values are represented as *probabilities*. A numerical example comprising one of the models is given in a later section to demonstrate the effectiveness of the criterion function methods.

Each class represents criterion-to-relative-weight function model; $a_i = f_j(x_i)$. In Willow and Ostrofsky (1997), criterion function methods were limited to Models I–IV. Here, Models V–VIII are introduced. These models are basically extensions to Models I–IV, respectively, in that (higher order) *criterion interactions* are included in the analysis. Fig. 15 depicts Model VI, a *linear* version of Model VII in Fig. 16.

Notice Model VII is a generalization of Model III. Moreover, it is a *continuous* (*viz.* non-linear) version of Model VI, in that it is generated as the number of intervals (within the criterion range) reaches *infinity* (∞). That is,

$$\lim_{\xi_i \rightarrow \infty} [a_i^v \cdot \mathbf{X}_i] \Rightarrow [a_i = g_i^v(\mathbf{X}_i)]; \quad v = 1, \dots, \xi_i. \quad (15)$$

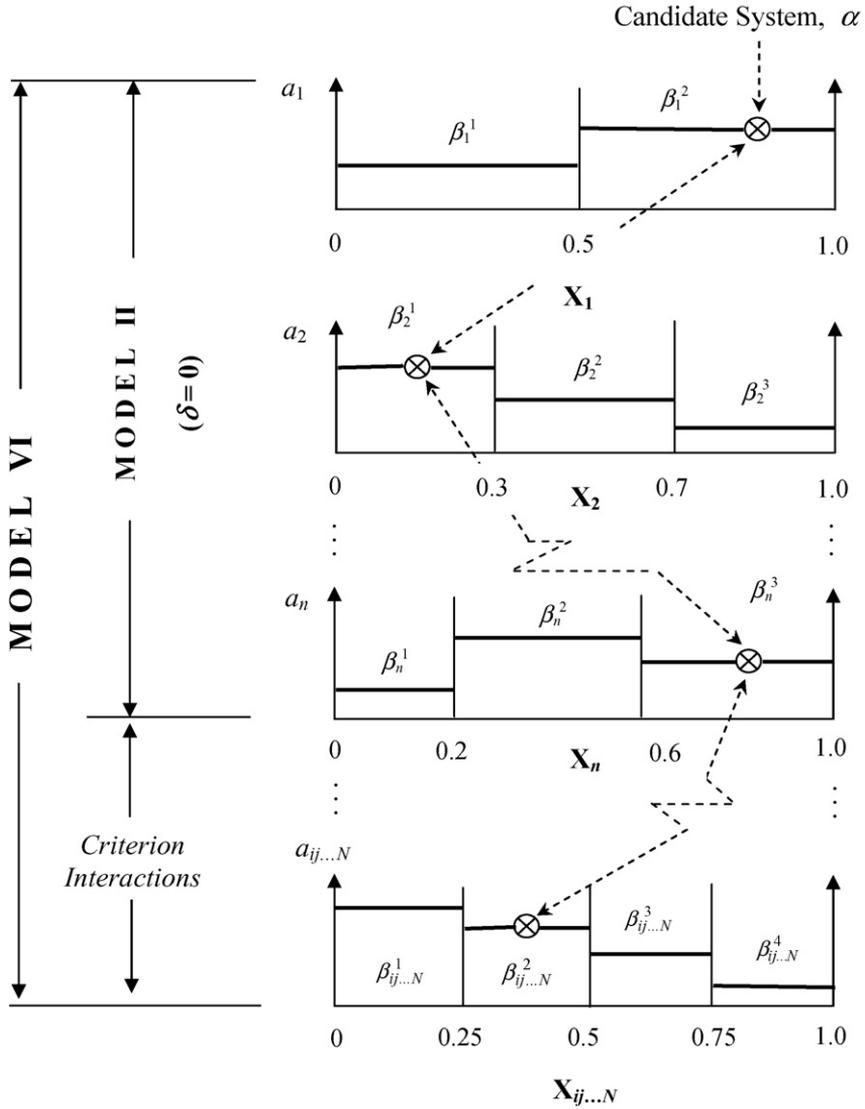


Fig. 15. Constant relative importance, mutually exclusive criteria with multiple intervals (Model II); interactive criteria with intervals (Model VI).

Note that a criterion value (x_i) for a candidate system, α , can be discovered at any point within the range of the criterion. Also note that the majority of engineering system design methodologies restrict their applications by exclusively using *uniform* relative importances for criteria (*i.e.*, Model I) and that by use of these criterion functions, *subjective* decisions could be made, which are explicitly embedded into the system modeling process. The criterion-to-relative-weight functions, in general, are determined and verified by management with top authority, thereby producing *qualitative* decisions.

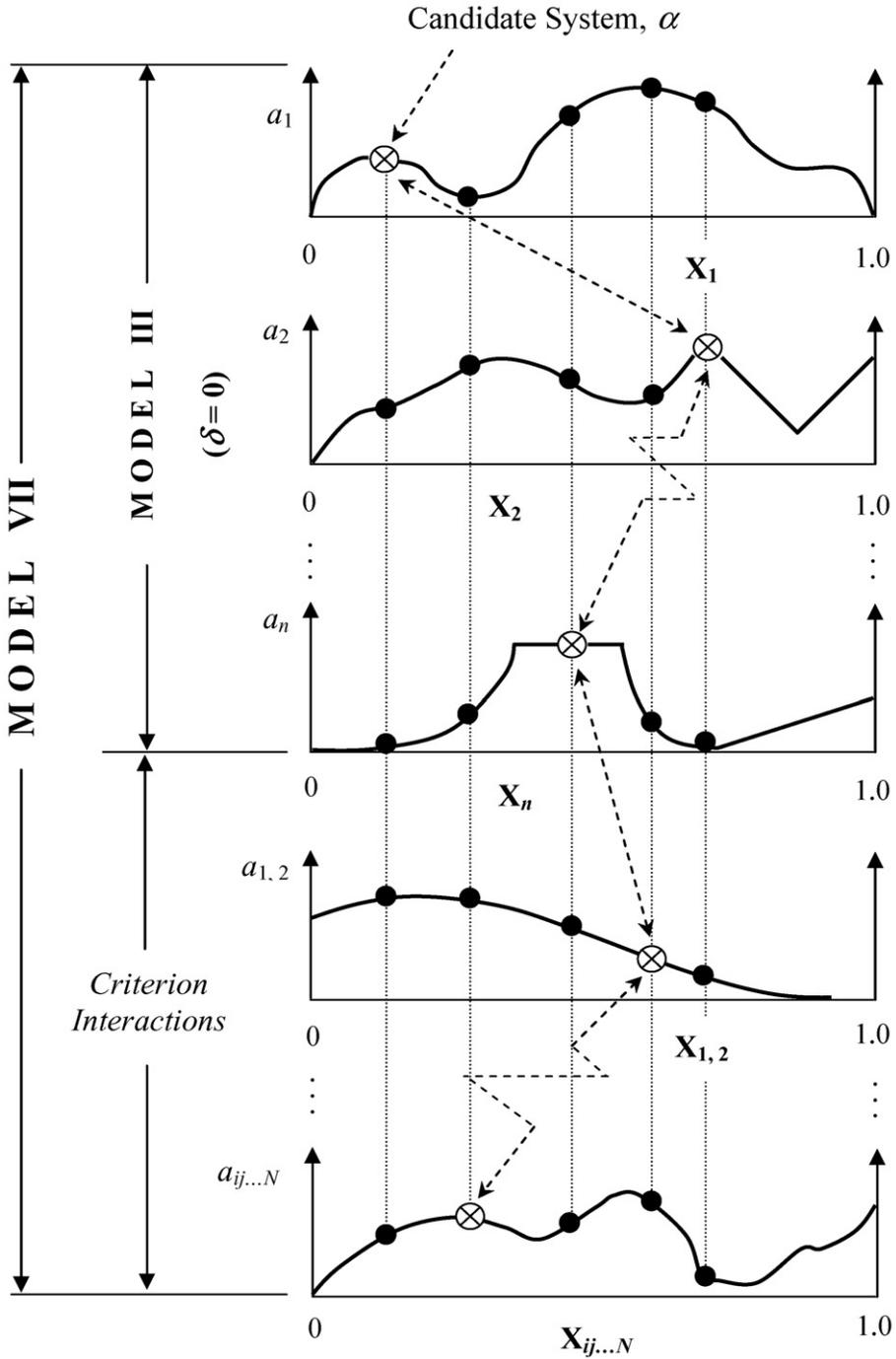


Fig. 16. Varying relative importance, mutually exclusive criteria (Model III); interactive criteria (Model VII).

5.1. General algorithm for criterion function methods

This section presents a general algorithm for selecting the *optimal candidate system* or alternative. A generalized procedure for *Model VI* is presented, since minor modifications (to it) could be made to retrofit the other models (*viz.* Models I–V, VII, and VIII). Readers are referenced to Willow (1999) for algorithmic computational procedures associated with other models.

As was the case for Model III among models with *mutually exclusive* criteria, models encompassing *interactive* design criteria are specialized variations of Model VII, and follows its algorithm accordingly. Moreover, Model VII is a generalization of Model III, thereby implying that it serves as a foundation for all other models. Here, however, Model VI, as a specialization (*i.e.*, *linear* version) of Model VII is described for simplicity. Following are the steps of its algorithm:

- (1) Map all x_i into \mathbf{X}_i (*i.e.*, normalize to unity).
- (2) Derive all *criterion intersections*, $\mathbf{X}_{ij\dots n}$.
- (3) Obtain the values of $a_i^{\beta_i^v}$ and $a_{ij\dots n}^{\beta_i^v}$ for all intervals, β_i^v .
- (4) Find the smallest *first* interval, B^1 , from among the \mathbf{X}_i and $\mathbf{X}_{ij\dots n}$. Note that B^1 accounts for the smallest fragment for all i , and should be distinguished from β_i^v .
- (5) Establish B^1 for all \mathbf{X}_i and $\mathbf{X}_{ij\dots n}$. (In Fig. 15, $B^1 = [0, 0.2)$ from \mathbf{X}_n .)
- (6) Establish the second overall interval, B^2 , from the end of B^1 to the next β_i^v limit. (In Fig. 15, $B^2 = [0.2, 0.25)$ from $\mathbf{X}_{ij\dots n}$.) Hence, B^2 extends to all \mathbf{X}_i and $\mathbf{X}_{ij\dots n}$.
- (7) Establish the next interval from the end of B^2 to the next β_i^v limit. Thus, $B^3 = [0.25, 0.3)$ from \mathbf{X}_2 for Fig. 15.
- (8) Repeat step 7 until the last interval, B^T , is reached. To verify T ($V = 1, \dots, T$), the following formula is employed:

$$T = \left(\sum_{i=1}^n \sum_{v=1}^{\xi_i} \beta_i^v \right) - (n - 1) \tag{16}$$

where n is the total number of criteria in the model and ξ_i is the number of intervals for criterion i . (For notation, refer to Section 3.)

- (9) *Vertical normalization among criteria (inter-criteria)*: normalize the a_i values in each interval B^V , so that

$$\sum_{i=1}^n a_i^{B^V} + \sum_{\substack{\forall i, j, \dots, N \\ i \neq j \\ \vdots \\ N - 1 \neq N}} a_{ij\dots n}^{B^V} = 1.0; \quad V = 1, \dots, T. \tag{17}$$

That is, we are *vertically* normalizing to unity, the a_i values for each (overall) interval, defined by B^V . Recall that V is the interval index across all \mathbf{X}_i and $\mathbf{X}_{ij\dots n}$. For instance, the relative importance for \mathbf{X}_1 in the first overall interval ($V = 1$) determined is defined by

$$a_1^{B^1} = \frac{a_1^{\beta_1^1}}{\sum_{\forall i} a_i^{B^1} + \sum_{i,j,\dots,n} a_{ij\dots n}^{B^1}} \tag{18}$$

- (10) Repeat step 9 for all $a_i^{B^1}$ and $a_{ij\dots n}^{B^1}$, and verify the values by using Eq. (17).

- (11) Repeat step 9 for all intervals, $V = 1, \dots, T$.
- (12) Identify all $a_i^{B^V}$ for each candidate system, α . That is, identify the $a_i^{B^V}$ and $a_{ij\dots n}^{B^V}$ corresponding to each \mathbf{X}_i and $\mathbf{X}_{ij\dots n}$, respectively, within a candidate.
- (13) *Horizontal normalization within a candidate (intra-candidate)*: for a given candidate system, α , normalize $a_i^{B^V}$ so that

$$a_i^* = \frac{a_i^{B^V}}{\sum_{\forall i} a_i^{B^V} + \sum_{\forall i,j,\dots,n} a_{ij\dots n}^{B^V}}, \tag{19}$$

or

$$a_{ij\dots n}^* = \frac{a_{ij\dots n}^{B^V}}{\sum_{\forall i} a_i^{B^V} + \sum_{\forall i,j,\dots,n} a_{ij\dots n}^{B^V}} \tag{20}$$

- (14) Normalize interaction terms, subject to the constraint $a_{ij\dots n} \geq a_{ij\dots(n-1)}$ (i.e., *conjunction rule*; see section B).
- (15) Compute for each candidate, α ,

$$CF_\alpha = \sum_{\forall i} a_i^* \cdot \mathbf{X}_i + \sum_{\forall i,j,\dots,n} a_{ij\dots n}^* \cdot \mathbf{X}_{ij\dots n} \tag{21}$$

Thus, the criterion function (CF_α) serves as a trade-off, evaluation function. The general form for the criterion function, CF_α follows in Eq. (22).

$$\begin{aligned} CF_\alpha &= Pr\left(\bigcup_{i=1}^n \theta_i\right) \\ &= \sum_{i=1}^n \theta_i - \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n \delta_{ij} \cdot \theta_{ij} : \text{ first order of interaction} \\ &+ \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n \sum_{\substack{k=1 \\ j \neq k \\ i \neq k}}^n \delta_{ijk} \cdot \theta_{ijk} : \text{ second order of interaction} \\ &\vdots \\ &+ \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n \dots \sum_{\substack{N \\ j \neq k \\ \vdots \\ N-1 \neq N}}^n \delta_{ijk\dots n} \cdot \theta_{ijk\dots n} : N = (n - 1)\text{th order of interaction} \end{aligned} \tag{22}$$

where

$$\delta = \left\{ \begin{array}{ll} 1, & \text{when } \exists \text{ Interaction} \\ 0, & \text{otherwise} \end{array} \right\} \text{ and } \theta_i = a_i \cdot \mathbf{X}_i.$$

The theory behind normalization, both vertical and horizontal, is the ‘axioms of probability’ (Casella and Berger, 1990). The purpose of the aforementioned algorithm was to develop a trade-off function, in order to determine the ordinal ranking of candidate systems on a *cardinal* (i.e., objective and uniform) scale. The cardinal scale was chosen to be the *probability* space, strictly ranging in the interval [0, 1]. Hence, normalized criterion values neither exceed 1 nor fall below 0. Notice that this applies to the criterion relative weights as well. Therefore, the relative weights and criterion values *equally* leverage and affect the CF in ranking and selecting the candidates.

A possible rank reversal exists when all criterion values for a particular candidate, α , are *equal*. Notwithstanding that this is true theoretically in the strict rigors of mathematics, returning to the issue of *engineering systems design*, the possibility of such an instance is almost *nil* in practice.

5.2. Intersection relative weights

Before proceeding, the notation used so far for the representation of relative weights has been somewhat loosely specified. Prior to this point, attempts have been made to remain consistent with the previous work being discussed. However, in this section, the following standard notation is utilized (Willow, 1999).

- a_i the relative utility weight of the *i*th criterion, as originally specified by the decision maker
- a'_i the *i*th relative weight following *vertical* normalization
- a''_i the *i*th relative weight following both *vertical* and *horizontal* normalization
- A_i the *i*th relative weight as finally included in the computation of CF_α

A second subscript may be used to denote the value for a specific candidate system and a superscript may be used to indicate the relative weight within a discrete interval of the X_i range. Note that under current practice, $A_i = a''_i$. An example is given in Table 1.

Since the relative weights are probabilities in their own right, the following ‘conjunction relation’ must be satisfied (Willow and Ostrofsky, 1997).

$$P(a_{ij}) \leq \min\{P(a_i), P(a_j)\} \tag{23}$$

Yet, as currently implemented, there is no assurance that this relationship will be satisfied. In order to avoid the violation, there might be a temptation to simply add a procedural constraint that Eq. (23) must be satisfied in the initial definition of relative weights. However, as would have

Table 1
Example relative weights

Criteria	Relative weights after vertical normalization (a'_i)
X_1	0.4
X_2	0.2
X_3	0.2
X_{12}	0.1
X_{13}	0.08
X_{23}	0.01
X_{123}	0.01
Sum	1.00

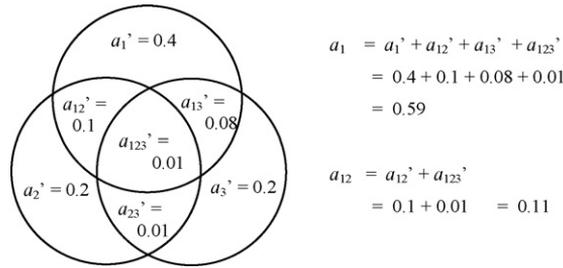


Fig. 17. Venn diagram for relative weights of Table 1 (after vertical normalization).

been the case with the functional interaction terms, such a stopgap effort would serve only to obscure an underlying source of inconsistency, which requires revision.

Reexamination of the Venn diagram in Fig. 17 reveals that, contrary to current practice, the marginal relative weights used in the computation of CF_α should not be as defined in Table 1. Rather, the complete marginal relative weight consists of the *sum* of the events that comprise it. That is, the relative weight of a marginal criterion (when placed into the criterion function) must include the relative weights of all intersections, of which it is a part. Similarly, the relative weight of an intersection term must include the relative weights of all *higher order* intersection terms, included within it. As a generic relationship, then, we state that

$$A_i = a_i'' + \sum_{\substack{j=1 \\ i \neq j}}^n a_{ij}'' + \sum_{\substack{j=1 \\ i \neq j \\ i \neq k \\ j \neq k}}^n \sum_{k=1}^n a_{ijk}'' + \dots + \sum_{\substack{j=1 \\ i \neq j \\ \vdots \\ N-1 \neq N}}^n \dots \sum_{N=1}^n a_{i\dots N}'' \tag{24}$$

Note that the realization in Eq. (24) infers that the a_i'' , as currently elicited and used, do not include the a_{ij}'' , as seen in Fig. 17. Thus, the a_{ij}'' are not currently the Boolean intersections of the a_i'' and a_j'' . However, with the additional operation of Eq. (24), the A_{ij} do, in fact, become the intersections of the A_i and A_j , satisfying the *conjunction rule* of (23).

The relationship leads to substantially improved insight regarding the meaning of the a_{ij} , as currently defined. Since \mathbf{X}_{ij} represents the measure of performance on criteria i and j simultaneously, the a_{ij} can be seen to represent the importance of a *balance* in performance among the intersecting criteria (possibly achieved at the expense of performance on marginal criterion values) (Willow, 1999). Since, for any particular application, no specific relationship may be assumed *a priori* concerning the importance of marginal criteria versus criteria balance, it is not appropriate to mandate any particular relationship between the a_i and the a_{ij} , other than that they sum to 1.0 (as achieved via *normalization*). Therefore, the a_{ij} may be either greater or less than the a_i . However, Eq. (24) is required to reflect the *hierarchical* relationship among intersection terms, which becomes necessary when relative weights are collectively placed in the context of the criterion function. In other words, (24) is the means by which the independently identified a_i and a_{ij} are made consistent with the laws of probability intersections and, hence, with the \mathbf{X}_i and \mathbf{X}_{ij} .

By the same token, relative importances of higher order can be deduced, based on the *conjunction rule*. For instance, a third-order relative weight is bounded below by zero and above

Table 2
Marginal and interactive criteria for the investment policy problem (example for Model VI)

Criteria and interactions	Candidate systems		$x_{i,\min}$	$x_{i,\max}$
	High interest, long-term savings account ($\alpha = 1$)	Stock and bonds ($\alpha = 2$)		
x_1 : initial capital (\$)	5000	5000	1000	10,000
x_2 : short-term interest (%)	12	5	3	100
x_3 : long-term, compound interest (%)	50	150	20	200
x_{12} (first-order interactions)	54	80	0	100
x_{13}	70	82	0	100
x_{23}	33	26	0	100
x_{123} (second-order interaction)	75	63	0	100

by the minimum weight of second order. Eq. (25) illustrates.

$$0 \leq a_{ijkl} \leq \min\{a_{ijk}, a_{ijl}, a_{ikl}, a_{jkl}\} \tag{25}$$

6. Numerical example

The following example illustrates an application of Model VI to a *financial system design* problem in the economics realm. The example is expected to be served as a proof that multiple criterion function methods of ISDM are applicable to a diverse area of generic systems design, encompassing engineering systems, as well as economic, business, manufacturing, and information. Currently, two alternatives or candidates exist ($\alpha = 1, 2$) in the selection for the most lucrative investment policy; opening a high interest, long-term savings account at a financial institution, or investing in stocks and bonds. Based on historical data, both marginal and interactive criterion values for each candidate, are as summarized in Table 2.

Normalized values (to unity) for the given criteria are obtained by using Eq. (26), the classical formula for linear interpolation.

$$\mathbf{X}_i = \frac{x_i - x_{i,\min}}{x_{i,\max} - x_{i,\min}} \tag{26}$$

In most cases, however, fitting the real-world data to a classical distribution, followed by a goodness-of-fit (GOF) test(s) are accomplished to obtain a *pdf* value, in lieu of Eq. (26) (Willow, 1999).

Thus, we have for all candidates,

• Candidate 1 ($\alpha = 1$)	• Candidate 2 ($\alpha = 2$)
$\mathbf{X}_1 = \frac{5000-1000}{10,000-1000} = \frac{4000}{9000} \cong 0.444$	$\mathbf{X}_1 = \frac{5000-1000}{9000} \cong 0.444$
$\mathbf{X}_2 = \frac{12-3}{100-3} = \frac{9}{97} \cong 0.093$	$\mathbf{X}_2 = \frac{5-3}{97} \cong 0.021$
$\mathbf{X}_3 = \frac{50-20}{200-20} = \frac{30}{180} \cong 0.167$	$\mathbf{X}_3 = \frac{150-20}{180} \cong 0.722$
$\mathbf{X}_{12} = \frac{54-0}{100-0} = 0.54$	$\mathbf{X}_{12} = \frac{80}{100} = 0.80$
$\mathbf{X}_{13} = \frac{70-0}{100-0} = 0.70$	$\mathbf{X}_{13} = \frac{82}{100} = 0.82$
$\mathbf{X}_{23} = \frac{33-0}{100-0} = 0.33$	$\mathbf{X}_{23} = \frac{26}{100} = 0.26$
$\mathbf{X}_{123} = \frac{75-0}{100-0} = 0.75$	$\mathbf{X}_{123} = \frac{63}{100} = 0.63$

Relative weights for marginal and interactive criteria were all assumed to be discrete, *linear* functions of the criteria (*viz.* Model VI) for clarity. Notice the identified set of criteria within a system may or may not be characterized by a single model, since they are inherently disparate. That is, one criterion might have the characteristics of Model IV, another Model II, yet another Model V, *etc.* Fig. 18 illustrates the functions of relative weights, verified by specialists and/or management.

Based on Fig. 18, Table 3 summarizes the X_i values, dispersed across the classes of data (*i.e.*, intervals).

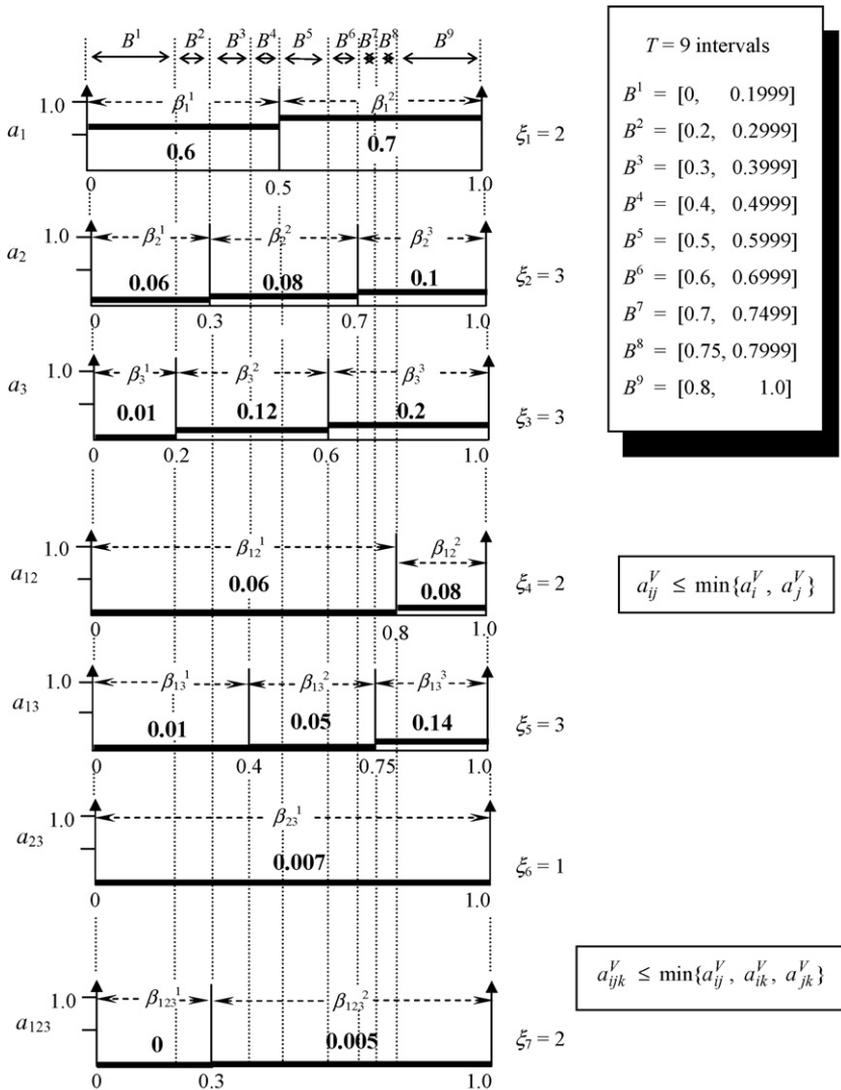


Fig. 18. Relative weights for marginal and interactive criteria (*Model VI example*).

Table 3
Normalized criteria within classes of intervals (example for Model VI)

Candidate systems (α)	Criterion values (X_i)	Intervals (B^V)								
		B^1 [0, 0.1999]	B^2 [0.2, 0.2999]	B^3 [0.3, 0.3999]	B^4 [0.4, 0.4999]	B^5 [0.5, 0.5999]	B^6 [0.6, 0.6999]	B^7 [0.7, 0.7499]	B^8 [0.75, 0.7999]	B^9 [0.8, 1.0]
1	X_1				0.444					
	X_2	0.093								
	X_3	0.167								
	X_{12}					0.54				
	X_{13}							0.70		
	X_{23}			0.33						
	X_{123}								0.75	
2	X_1			0.444						
	X_2	0.021								
	X_3							0.722		
	X_{12}									0.80
	X_{13}									0.82
	X_{23}		0.26							
	X_{123}						0.63			

- *Vertical normalization* (across criteria i , within an interval V): computations for the first interval (B^1) are given to illustrate.

(1) Marginal relative weights:

$$a_1^{B^1} = \frac{a_1^{\beta_1}}{\sum_{\forall i} a_i^{B^1}} = \frac{0.6}{(0.6 + 0.06 + 0.01) + (0.06 + 0.01 + 0.007) + 0} = \frac{0.6}{0.75} = 0.8;$$

$$a_2^{B^1} = \frac{0.06}{0.75} = 0.08;$$

$$a_3^{B^1} = \frac{0.01}{0.75} = 0.013$$

(2) First-order interaction weights:

$$a_{12}^{B^1} = \frac{0.06}{0.75} = 0.08;$$

$$a_{13}^{B^1} = \frac{0.01}{0.75} = 0.013;$$

$$a_{23}^{B^1} = \frac{0.007}{0.75} = 0.009$$

(3) Second-order interaction weights:

$$a_{123}^{B^1} = \frac{0.0}{0.75} = 0$$

Vertically normalized relative weights for $V = 2, \dots, T$ are tabulated in Table 4 that follows. Notice that vertical normalization need not be performed for relative weights across the entire classes of criterion intervals or \mathbf{X}_i (or $\mathbf{X}_{ij \dots N}$) values. In other words, it suffices to accomplish vertical normalization for only those relative importances which are associated with the normalized criteria. Table 4 was prepared to provide a complete listing and to emphasize that the sum of each column, thus for vertically normalized weights, is indeed one (*i.e.*, unity). Note also that the *conjunction rule* in Eq. (23) has to be satisfied for relative weights of all orders of interactions.

- *Criterion function for alternative selection with horizontal normalization*: recall that the equation for criterion function (CF) is

$$CF_\alpha = \sum_{i=1}^n a_i^* \cdot \mathbf{X}_i - \sum_{\forall i} \sum_{\forall j} a_{ij} \cdot \mathbf{X}_{ij} + \dots (-1)^{N-1} \sum_{\forall i} \sum_{\forall j} \dots \sum_{\forall N} a_{ij \dots n}^* \cdot \mathbf{X}_{ij \dots n}, \tag{27}$$

with

$$a_i^* = \frac{a_i^{B^V}}{\sum_{\forall i} a_i^{B^V} + \sum_{\forall N} a_{ij \dots n}^{B^V}} \tag{28}$$

and

$$a_{ij \dots n}^* = \frac{a_{ij \dots n}^{B^V}}{\sum_{\forall i} a_i^{B^V} + \sum_{\forall N} a_{ij \dots n}^{B^V}} \tag{29}$$

Table 4
Vertically normalized relative weights within criterion intervals (example for Model VI)

Relative weights (a_i)	Intervals (B^V)								
	B^1 [0, 0.1999]	B^2 [0.2, 0.2999]	B^3 [0.3, 0.3999]	B^4 [0.4, 0.4999]	B^5 [0.5, 0.5999]	B^6 [0.6, 0.6999]	B^7 [0.7, 0.7499]	B^8 [0.75, 0.7999]	B^9 [0.8, 1.0]
a_1	0.800	0.700	0.680	0.651 ^{a,b}	0.685	0.635	0.624	0.578	0.568
a_2	0.080 ^{a,b}	0.070	0.091	0.087	0.078	0.073	0.089	0.083	0.081
a_3	0.013 ^a	0.140	0.136	0.130	0.117	0.181	0.178 ^b	0.165	0.162
a_{12}	0.080	0.070	0.068	0.065	0.059 ^a	0.054	0.053	0.050	0.065 ^b
a_{13}	0.013	0.012	0.011	0.054	0.049	0.045	0.045 ^a	0.116	0.114 ^b
a_{23}	0.009	0.008 ^b	0.008 ^a	0.008	0.007	0.006	0.006	0.006	0.006
a_{123}	0.000	0.000	0.006	0.005	0.005	0.005 ^b	0.004	0.004 ^a	0.004
$\sum a_i$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

^a Alternative 1 ($\alpha = 1$).

^b Alternative 2 ($\alpha = 2$).

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